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DEVELOPING A CONCEPT MAPPING APPROACH TO MATHEMATICS ACHIEVEMENT IN MIDDLE SCHOOL

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Abstract. Concept mapping has been very useful in a variety of disciplines; however, there has been relatively little use of it in mathematics in the United States. A project to bring the use of concept mapping into middle grades mathematics was begun at the University of North Florida (UNF) in the spring of 2005, and has produced a design for a two-week, summer middle grades mathematics camp for delivery in the summer of 2006. Preceding the camp is a series of teacher professional development workshops provided for the (middle school) teachers of the camps. The workshops focus on the use of concept maps in planning, classroom delivery, and assessment; on the use of the CmapTools software; and, on the development of activities appropriate to Algebra I topics or readiness, and also which integrate a concept mapping component. This paper describes the development process, the instruments developed, the summer camp itself, and planned follow-up during the 2006-2007 school year.

1 Introduction and Overview

In early spring 2005, the Florida Institute of Education at the Unversity of North Florida (FIE) joined in a partnership with the Institute of Human and Machine Cognition (IHMC) with the goal of bringing the use of concept mapping into middle grades mathematics. FIE was already engaged in a middle grades mathematics enhancement program called the PreCollegiate Program. The partnership with IHMC was formed to bring concept mapping into that program through the use of CmapTools[1] software.

A team of three UNF mathematics faculty members, two high school teachers experienced in teaching Algebra I, and two FIE professionals comprised the development team (hereinafter, the team), and during the period May-July, 2005, IHMC provided the team with extensive training in the use of CmapTools. Throughout the remainder of the calendar year, the team focused on designing and writing a set of expert maps describing Algebra I as a connected set of mathematical concepts. The following protocol was established by the team: *In every expert map produced, any 3-element sequence of concept-connector-concept must stand alone as a correct and mathematically meaningful sentence.*

During the spring of 2006, the team worked with two independent Florida School districts, Polk County Schools, and Duval County Schools (in Florida, each county represents a single school district) to plan summer mathematics camps for high-risk middle school students from low-performing middle schools in each district. This planning was accomplished through a series of four teacher-development workshops and several follow-up planning sessions. The workshops were conducted by the team, and were open to all middle school mathematics (and science) teachers from the participating schools in each district.

The summer camps had three primary objectives:

i. to increase the interest of the participating students in learning about mathematics and its applications;
ii. to improve the readiness of the students for entering and succeeding in Algebra I; and,
iii. to introduce concept mapping, through CmapTools, as a mathematics teaching, assessment, and learning strategy.

Four full-day workshops were designed to have the teachers learn to use CmapTools and also to have the teachers develop well-organized mathematics and applied mathematics activities that were designed to

i. increase student interest in learning mathematics;
ii. focus on well-defined mathematics objectives that were related to Algebra I; and,
iii. introduce concept mapping as a component of the teaching and learning strategy.

[1] http://cmap.ihmc.us

The summer camps were designed in April and May, 2006, and then delivered in June.

2 Concept Mapping of Algebra I

The approach taken to defining Algebra I for mapping purposes was to first decide on the major concepts to be featured. The major concepts selected by the team were:

Equations and their solutions
Inequalities and their solutions
Concepts associated with exponents
Polynomials and rational expressions
Functions
Graphing

An expert map connecting these major concepts, related sub-concepts, and essential prerequisites was developed by the team in the fall of 2005, and modified as the project developed over the spring 2006 term. The current Algebra I master map is given in Figure 1.

Each of the sub-concepts identified in the master map was mapped by the team, and was connected through CmapTools to the master. Each map created was revised many times before reaching its current state. These maps are expert maps, and are not intended as fundamental teaching tools to be read by the students. Rather, they are for review and reference by the teachers as they examine their individual understanding of the mathematical concepts associated with Algebra I and the connections among those concepts.

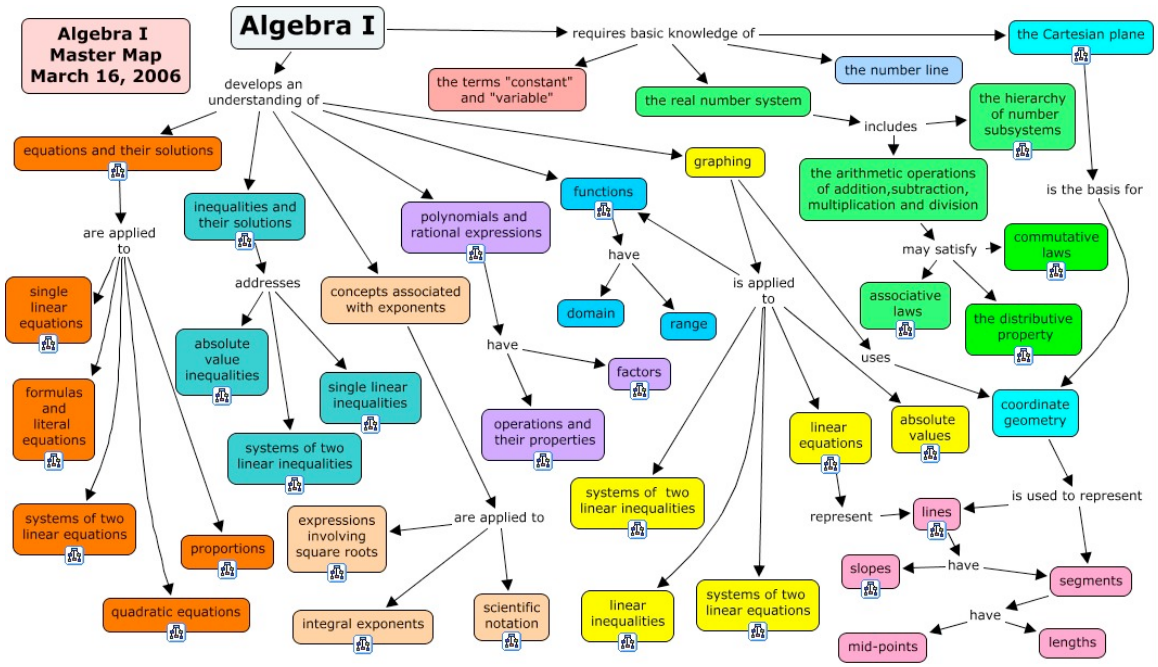

Figure 1. Algebra I Master map.

The protocol established by the team was that any sequence of concept-connector-concept should read independently as a complete, correct, and meaningful sentence. This restriction was adhered to as much as possible, and sometimes led to maps that were somewhat more complex than was hoped for by the team. An example is given in Figure 2.

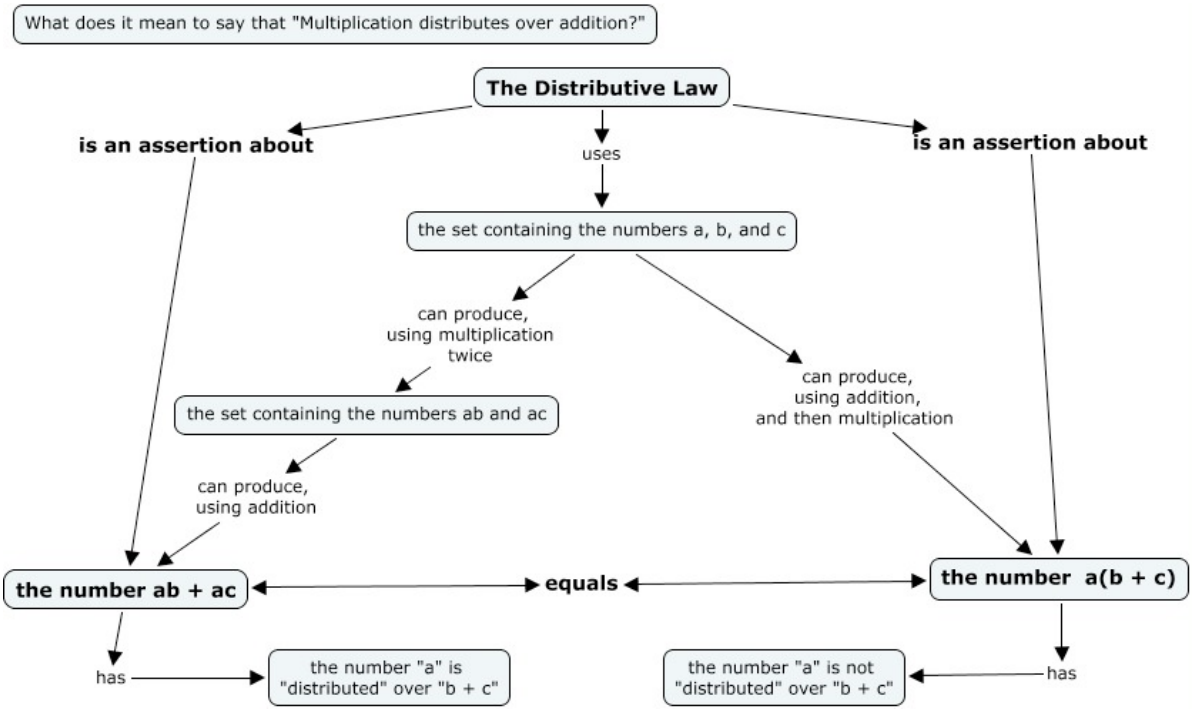

Figure 2.

Despite this possible consequence, the team felt that adherence to the protocol forced one to think carefully and correctly about the concepts that were being connected, and also what was really the most appropriate connection, so the protocol was maintained.

While developing the Algebra I expert maps, the team observed that a mathematics concept map node had three possible kinds of objects that could be connected to it to provide useful information about, and understanding of, the concept. These were: other concept maps; examples of things that satisfied or described the concept; and, in some cases, how to perform processes associated with the concept. The last of these, processes, lent itself easily to being described through the CmapTools software, and the team developed some important process maps to be connected to the master and other maps, as clarifying objects.

3 The Workshops

The four teacher development workshops each were for a full day, and each included: discussion of concept mapping; time for using CmapTools; work on a typical summer camp activity; and a discussion of the activity's mathematical focus. Figure 3 gives a typical workshop schedule.

Polk County / FIE Summer Camp Workshop III **Boone Middle School** **Agenda** **March 11th, 2006**	
8:00 – 9:30	Large number activities: *Arranging Books* *Drips of Water*

9:30 – 10:00	Discussion of the number sense activities
10:00-11:45	Teams build concept maps of large numbers – A planning application
11:45 – 12:00	Discussion: Summer Camp 2006
12:00 – 12:30	Lunch
12:30 – 1:30	Review of an activity from last year's summer camp *Packaging and the Environment*
1:30-2:45	CmapTools work – Connecting the mathematical concepts in a chapter of a typical text
2:45 – 3:00	Evaluation

Figure 3.

In one workshop, the team presented the teachers a concept map constructed for Unit 2 of the CPM Algebra I text (Figure 4). Afterward, the teachers constructed a concept map of all basic mathematical concepts in a single chapter of a text they currently used in one of their classes.

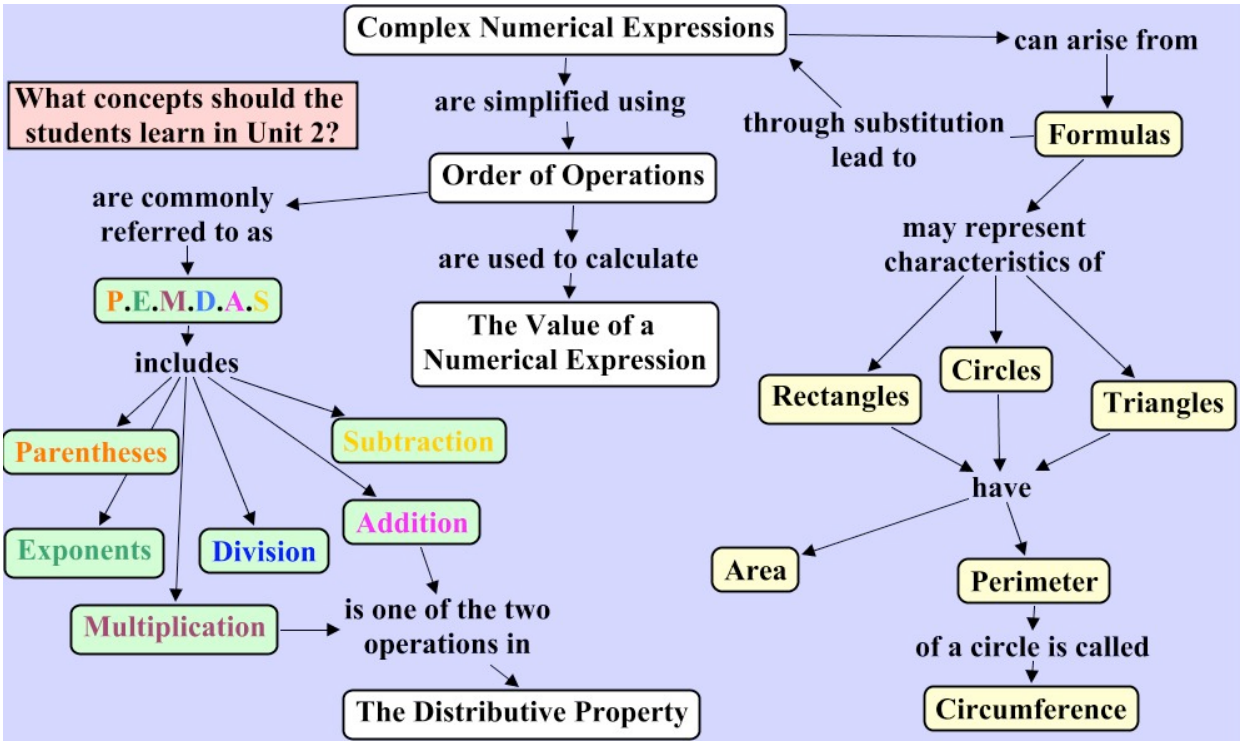

Figure 4.

When asked how they could help students appreciate the size of 1 million, the teachers came up with some interesting process maps, one of which is given in Figure 5.

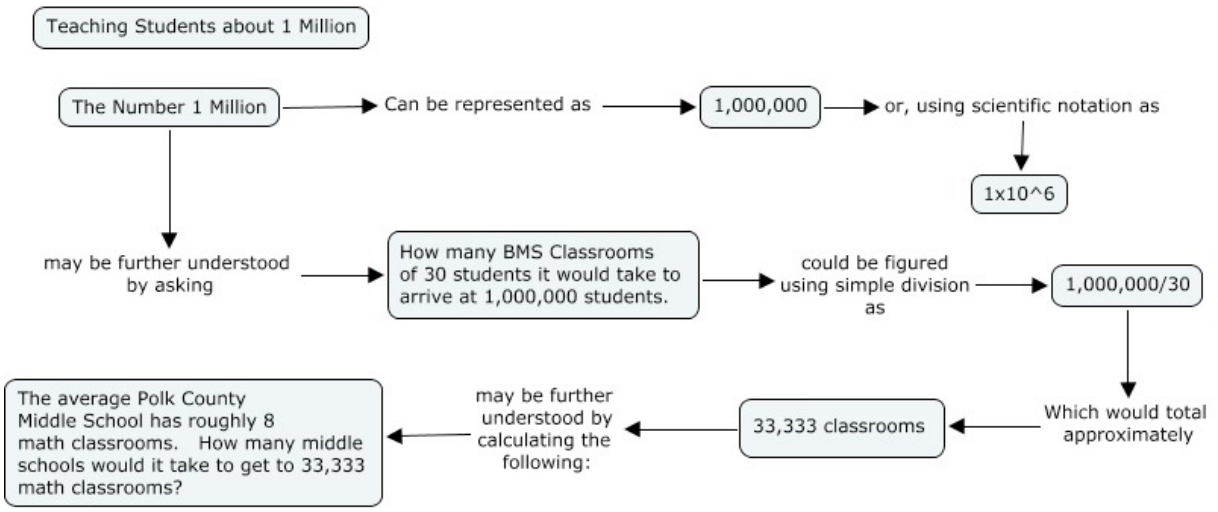

Figure 5.

4 The Summer Camps

The summer camps were developed from a bank of activities that were produced by teachers in the PreCollegiate Program over the years of its operation. A typical activity is *Timing the Circle.*

"Timing the Circle"

Activity Description:

A 40-foot diameter circle is laid out in an open field, and a diameter is indicated. Starting at one end of that diameter, a student walks at a steady pace around the full circle. The time elapsed for the circuit is recorded. Then, the same student walks at the same steady pace from one end of the diameter to the other, again recording the elapsed time. Once all students in the class have performed this same activity, each student calculates the quotient of his/her time to go around with time to go across the circle. The students than compare the numbers they have calculated. The experiment is repeated for a different circle. If a student's individual speed has been uniform for walking the diameter and walking the circumference, the numbers calculated will be reasonable estimates for π.

Mathematical Vocabulary Introduced:

Circle
Center of a circle, O
Diameter of a circle, D
Circumference of a circle, C
Radius of a circle, R
The number π

Mathematical Relationship Discussed:

$C = \pi D = 2\pi R$

Physics Formula Discussed:

Distance = Rate x Time

Mathematical Goals of the Activity:

1. To have the students understand basic properties of circles.
2. To have the students discover that the ratio of the circumference to the diameter of a circle is constant, and to estimate what that constant is.

3. To introduce to the students a relationship between physics and mathematics.

Equipment Required:

If circles cannot be marked out using a standard sports-field line marker, materials to make them will be required. Construction tape or marking flags can be used.

Stopwatches
Tablet/pencils
Calculators

Conducting the Activity:

A discussion of properties of circles should take place either in class on the day of the activity, or on the field immediately before the activity begins. A table with each student's name and columns for entering times across and times around should be prepared in advance. The circles also should be set up in advance. Once on the field and ready to start the activity,

1. Discuss what will take place, and emphasize the importance of keeping a steady pace and of using the same pace going across as around the circles.

2. Assign some students to be timers, and some to be recorders. Two timers should be used for each student, and a walk repeated if the two stopwatch times are too far apart.

3. Have the students walk around a circle then across the circle at the same pace, timing the walks and recording their results on the table. If there is enough time, have each student walk it twice, at different rates of speed.

4. Once all students have walked, calculate the time around divided by the time across for each row of the table.

5. Get everyone involved in calculating the quotients. The calculations can be done on the field or back in the classroom, but should be done so that everyone can compare their own numbers with those of others. Calculate a class average and compare it with π.

6. Discuss how the quotient of two *times* can give the same thing as the quotient of the two *distances* – the circumference and the diameter.

7. Have students complete worksheets giving some practice with calculating circumferences (e.g., calculating hat size, or showing what happens to the circumference when changes are made in the diameter).

8. Use the Vocabulary List to have students prepare a concept map with the circle as the basic concept.

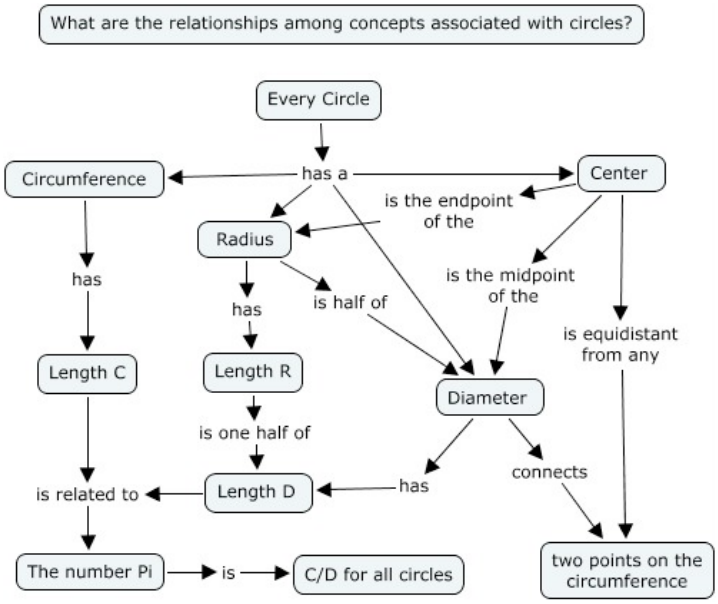

Figure 6: Possible Expert Map for *Timing the Circle*.

Each activity is organized in a fashion that states clearly the mathematical objectives of the activity, the connection of those objectives to appropriate standards, a method for carrying out the activity, the basic mathematical concepts involved, and a concept map of those ideas. Camps were scheduled for two, five-day weeks, and each included 35-40 students. Each camp was conducted by middle school teachers, with several team members acting as consultants each day. Both the teachers and the students received stipends for participating in the camps. A pre/post test was conducted to indicate the impact of the camp on the students.

5 Project Continuation, Fall 2006

The primary purpose of the project is to have impact on the students' performances in mathematics, and to encourage them to apply themselves more to their mathematics courses. To keep this encouragement active, the teachers from the camps will introduce concept mapping into their regular classrooms beginning in the fall of 2006. The FIE Team will visit classes and will consult with the teachers over the course of the year, and the FCAT[2] results from 2006 to 2007 for participating students will be compared with the change for non-camp students with comparable scores in 2006, to see if there was any impact from the camp and the subsequent concept mapping instruction.

The Algebra I mapping project will be completed in the fall of 2006, and a series of professional development workshops based upon those expert maps will be developed for delivery at several sites throughout Florida in the spring of 2007.

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[2] * The Florida Comprehensive Assessment Test (FCAT) is the official measure of student performance administered by the State of Florida each spring.