Concept Maps: Theory, Methodology, Technology
Proc. of the Second Int. Conference on Concept Mapping
San José, Costa Rica, 2006

COMPREHENSION, ANALYSIS AND DERIVING MEANING

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Abstract. The versatility of the concept map is utilized in a current Teaching-Research project by (1) teacher-researchers for analyzing student's calculus essays, (2) students to create a schema from scattered concepts in developmental mathematics classes and (3) web designers for clarity of presenting intricately connected concepts. In all three cases, the concept map is the central conceptual structure that motivates learning.

1 Introduction

The work described in this paper is a part of, or an outgrowth of the project, Introducing Indivisibles in Calculus Instruction, NSF-ROLE #0126141 whose principal aim was to investigate student understanding of the definite integral in Freshmen Calculus. This project utilized the Teaching-Research Methodology (TR-NYC model) as its primary modus operandi. TR-NYC is a careful integration of the principles of Action Research with the principles of Vygotsky's Teaching Experiment, used to assess "the changes in mental structures of students under an impact of instruction". This integration allows for the bi-directional approach to Teaching-Research, from research to practice and from practice to research.

The concept map serves a variety of roles in the hands of a Teacher- Researcher: a medium of communication between members of teams, (including diverse student populations) where some situations require a de-emphasis of the written language in favor of visual structural communication; it is an instructional tool to stabilize the conceptual understanding of the mathematical situation; finally a concept map can serve as the precise assessment tool for written mathematical essays.

2 Communicating Knowledge via the web using concept maps

Concept mapping has been promoted as an educational tool that adds quality to the learning process providing individuals with the means for making meaning of complex knowledge, in their own way. Since concept maps are graphical tools that include relationships between concepts, combining this tool with the medium of the internet provides for wider collaboration. The visual representation of our ways of thinking about teaching, learning and research are made transparent to those who are involved in the projects as well as others. In the future, concept mapping is envisioned as a tool for representing one's own learning and linking multiple visual products to the web to create a community of learners who are separated geographically. The poster presentation will provide the concept maps that detail our ideas.

3 Building a schema in Developmental Mathematics

Development of the schema is an individual process and can only be speeded up or enhanced by the learner. However, having access to certain synthesizing, differentiating, and connecting tools such as the concept map provides the learner with a language, a medium for learning. The concept map is in some sense a "seed", a gestalt (Korthagen and Kessel, 1999) which can grow to create the schema of the course and which by the nature of its pictorial representation, reveals to the instructor the areas where the student does, and does not have a grasp of the concept in question. Students on the other hand, have access to the concept map as an organizing principle.

In the context of the area and perimeter problem, (prior to the use of the concept map as a learning medium), it was found that students tended to ignore parts of the problem stating they "did not see" the ignored part. As a result only 2 (out of 33) students solved the problem correctly. Apart from ignoring parts of the problem, the instructor diagnosed that students do not have a schema for the concept of area and perimeter, that they tend to "remember" sometimes incorrectly, even the formulas for area and perimeter and hence were not even equipped with the bare essentials needed to attempt the problem. The concept map was introduced to the class in this situation to be used in

three stages: (i) the map is constructed for the geometric object in a given problem but without using any other information of the problem, (ii) inserting on the concept map the information contained in the problem (in blue below), (iii) translating the inserted information into mathematical language for problem solving (in red below).

Thus, for the problem:

> *The length of a rectangle is four times its width. The area of the rectangle is 88 sq. ft. Find the perimeter of the rectangle.*

The following concept map was introduced to the class in the three stages described above. Students then tried the concept mapping technique for themselves in 2 in-class problems and 1 homework assignment.

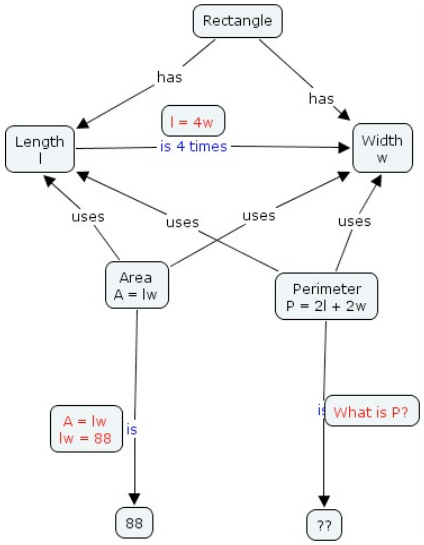

Figure 1. Concept map constructed in three stages

Methodology and Preliminary findings: Only 1 whole class period was devoted to using concept maps to the topic of finding area and perimeter of different geometric figures. The concept of straight lines that followed was "taught" without concept maps and students were asked to construct a concept map of the 5 connected concepts. Only some students returned the assignment. The concept map was never "indoctrinated". On quizzes and tests that followed the first use of the concept map, students often asked the instructor whether they should be using the concept map and were told, that the decision was theirs. It could be used if needed. The following shows the concept map of a student (Student C) who consistently used it in the following two tests.

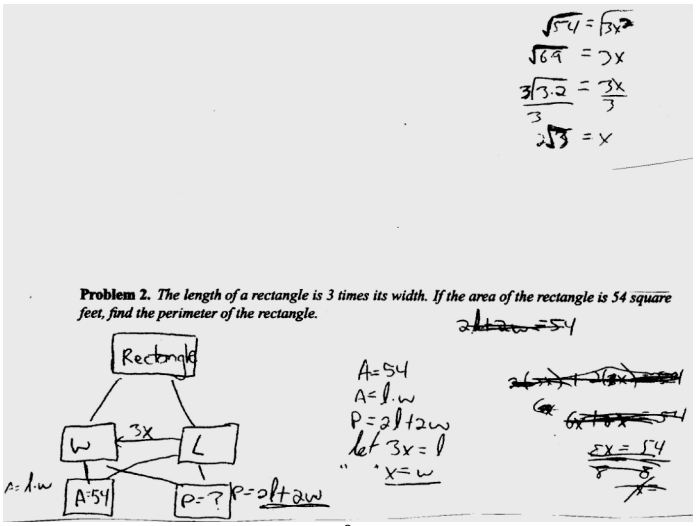

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Figure 2. Concept map of Student C

Student C has drawn in both instances a concept map, what indicates correct understanding of the situation, correct formulas and their meaning. Observing the solution, we see now better what stops the student in his tracks; it is arithmetic and algebraic errors rather than the absence of understanding, in fact, is the third step amongst the three through which the instructional potential of the approach is assessed. By introducing the concept map we were able to separate different components of understanding in the student mind and focus on the real problems with computations. For a student too, it is difficult to say "I didn't see" once he/she has to build the corresponding concept map from scratch. Hence, we conclude that the issue is not in "not seeing" the components of a problem but in the absence of basic skills, we see there is a break in student mind between understanding the structure of the problem through the concept map and solving the problem on this basis. Quite possibly, the next re-design done in the context of the cyclic TR methodology needs to develop the concept map incorporating the steps of the problem solving approach.

It is interesting for students themselves to see their own "schema" of the concept in discussions with the instructor and the direct correlation of the incomplete information in the concept map and the corresponding error in the computations of the problem. It is through seeing this correlation, that students themselves learn the value of the concept map and its effect on their thinking. Furthermore, a scattered schema is evident from the concept map as in the case of Student E below.

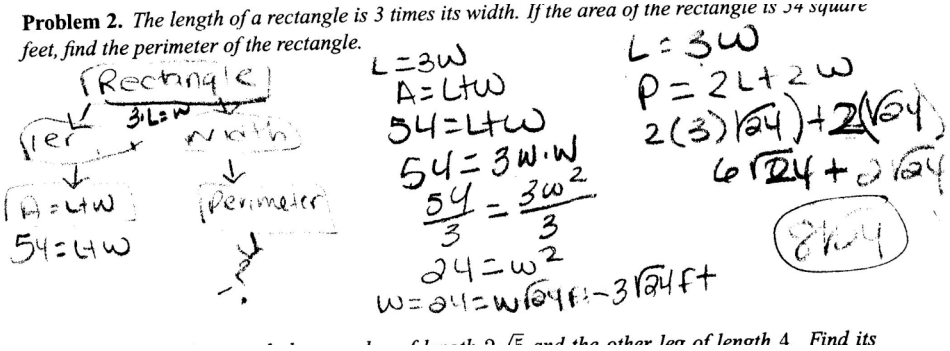

Figure 3: Concept map constructed by Student E

In general, the introduction of this particular concept map proved to be an effective tool. Whereas pre-test examination revealed only 2 students out of 33 being able to do the problem, a post test exam revealed 21 out of 33 who grasped the concept. At the same time we note the limitation of the tool in addressing the computational difficulties of students.

4 Analysis of Student Mathematical Essays in Calculus:

Reading student mathematical essays is not easy, yet it provides significant information about student thinking of the mathematics in question. A mathematical essay is difficult to assess precisely because of the absence of symbolic notation, which gets converted into sentences with their compact structure. An essay is traditionally assessed by a multidimensional rubric of some type, where the simultaneous gradation of each dimension provides the scale of the instrument. For a teacher-researcher the student essay is not only the assignment allowing the student freedom of written expression, which according to Luria and Yudowich (1971) "assumes slower, repeated mediating process of analysis and synthesis" but also is the instrument of investigation providing the information about the nature, extent and precision of student thinking through the processes of analysis and synthesis. The rubric, in such a case, is too weak of an instrument of assessment. It cannot tell the investigator precisely where are the essential problems of student understanding of the limit of an infinite sequence. But the concept map can. We take as the nodes, the essential concepts taking part in the construction of understanding, and, through the concept map, we seek to map out the relationships we find in student thinking about the problem. We map out every sentence that conveys information about the involved concepts as nodes connecting arrows. The structure of the arrows gives us a strong indication of the degree of student understanding. Sentences in the essay can indicate that contradictory processes are taking place in the student mind. Contradictory sentences which one often meets within one essay become understood as indicating weak spots in the emerging schema of the concept for the particular student. More often

then not, the problematic areas indicate a fundamental logical problems, or a misconception characteristic for calculus student in U.S. hence the design of corrective intervention follows the discovery of the learning problem, completing the cycle of Teaching-Research methodology. The poster presentation will provide the concept maps that provide assessment of student work.

5 Summary

The concept map is a learning medium. It is a medium in which the students can learn new concepts and discover what they are missing in their schema, and with this discovery, they can enhance their own learning. The teacher-researchers can learn to communicate. The web designers task is perhaps the most pivotal. He has to know which tools are appropriate for the idea that is not his, and for this, he has to discover the entire concept for himself. When the concept map is used for this discovery rather than language, the web designer has access to the components that make up the schema and so he begins building the schema and he does so by fitting the tools he or she thinks right. In the ensuing communication negotiation of meanings results till both have their own schema represented in the map. The map thus becomes the medium of learning for both parties and the shared meaning is represented in the concept map, allowing the readers to construct their own schema and to learn. The teacher-researchers learn the schema of their students and in the process learn about the different interpretations of their own teaching.

Why is the concept map a learning medium? It provokes thought, and hence the thinking of the person in question is set in motion, i.e., it changes state from the inertia to one of movement, and once in motion it is free to go in the direction chosen by the learner. The concept map is an emerging tool for the building of the schema in the project and as such the results based on the use of the concept map are preliminary in nature. The key in the use of the concept map is to find the central conceptual structure that lies at the heart of the concept in question. The central conceptual structure for the concept of fractions is the Fractions Grid (Czarnocha & Prabhu, 2005) which is entirely iconic/concrete. The concept map has language though a limited amount, and becomes a medium between the concrete rectangle/square/circle/trapezoid/triangle and the abstract "word problem". Problem solving skills along with the schema of the geometric figure both play a part in the successful solution. It was found that the concept map helps with the building of the schema, and the teaching-research question arises, how can it enhance problem-solving?

6 Acknowledgements

This Research Project was partially supported by Grant #0126141 from the National Science Foundation.

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