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INNOVATIVELY DEVELOPING A TEACHING SEQUENCE USING CONCEPT MAPS

Karoline Afamasaga-Fuata'i, University of New England, Australia
Email: kafamasa@une.edu.au

Abstract. A student teacher's work with concept maps illustrates the conceptual structure underpinning a teaching sequence in order to communicate efficiently his perceptions of what it means to developmentally and conceptually teach a selected topic in contrast to simply compiling a sequential list of sub-topics. Main insights from the case study were that constructing concept maps prompted the student teacher to reflect deeply about his own knowledge of mathematics beyond the assignment topic and challenged him to strategically organize the results of this thinking and conceptual analysis into visual, hierarchical displays of concept networks to parsimoniously and meaningfully illustrate the relationships between key and subsidiary concepts of the assignment topic.

1 Introduction

Whilst syllabus outcomes and key ideas are useful to guide the planning of teaching sequences, "they are only 'frameworks' – teachers need in-depth knowledge of mathematical concepts and processes so as to enrich them" (Bobis, Mulligan & Lowrie, 2004, p. 25). With the prevailing curricular emphasis on encouraging students to think mathematically (New South Wales Board of Studies (NSW BOS), 2002)), there is a need to conduct research into innovative ways of supporting mathematical thinking and reasoning in deeper and more conceptually based ways. Hence, the main study explored ways in which growth in understanding mathematical concepts and processes could be supported through concept mapping and vee diagramming by investigating secondary student teachers' use of concept maps and vee diagrams (i) to critically analyse selected content of the *7-12 Mathematics Syllabus* (NSW BOS, 2002), (ii) to illustrate their conceptual understanding of syllabus outcomes and indicators, activities and problems; and (iii) to develop requisite skills in the design of conceptually rich activities to promote working and communicating mathematically. The study was guided by Ausubel's theory of meaningful learning which proposes that learners' cognitive structures are hierarchically organized with more general, superordinate concepts subsuming less general and more specific concepts (Ausubel, 2000; Novak, 2004). By constructing concept maps and vee diagrams (maps/diagrams), students illustrate publicly their interpretation and understanding of a topic/problem in terms of interconnections between concepts, principles and methods. Recent research (Afamasaga-Fuata'i, 2005, 2004a, 2004b) with Samoan undergraduate mathematics students demonstrated the usefulness of maps/diagrams as valuable meta-cognitive tools to scaffold students' thinking and reasoning, to illustrate students' developmental and conceptual understanding, and to enhance efficiency in communicating mathematically as they learnt new mathematics topics or solved mathematics problems in their university mathematics courses. Through participation in social critiques over the semester, students received constructive feedback to further improve individually constructed maps/diagrams; subsequently their end-of-study maps of assigned topics were structurally more complex and differentiated than initial maps as a result of thinking about thinking, interactions with others and concept mapping. Whilst these studies (Afamasaga-Fuata'i, 2005, 2004a, 2004b) focused on undergraduate students' applications of maps/diagrams as learning, the main study that is partially reported here with student teachers at an Australian regional university focused on the applications of maps/diagrams as pedagogical tools. The following sections briefly describe the study's methodology before presenting data from one student teacher's work in developing a teaching sequence through concept mapping. Discussion of the student teacher's concept map data and some insights based on the case study are also provided.

2 Methodology & Data Collected

The main study's methodology was a design experiment in which student teachers critically analysed syllabus outcomes, problems and activities (i.e. *critical analysis*) for underlying concepts and principles (i.e. *conceptual structure*) before illustrating the results on maps/diagrams followed by an examination of (a) the kinds of *discourse* that emerged during *critiques* of presented maps/diagrams and *student reflections* on how their constructing experiences impacted on the way they planned, thought and viewed the teaching of mathematics topics; (b) the types of participation norms (i.e. *socio-mathematical norms*) established for the development and critique of maps/diagrams during weekly workshops; and (c) the types of *practical means* by which the researcher "orchestrated relations among these elements" (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003, p. 9). The sample included ten internal students of the two secondary mathematics education units who agreed to participate. The lecturer-researcher introduced and used maps/diagrams in her presentations of materials during weekly workshops. Regular assignments, in parts, required students to prepare unit plans and lesson plans for various content areas of the *NSW 7-12 Mathematics Syllabus*. The study was in two parts. Firstly, as learners, students constructed maps/diagrams as tools to illustrate and communicate their conceptual and methodological understanding of the mathematics content in activities/problems. Secondly, as student teachers preparing for teaching practicum, they developed lesson plans and activities using maps/diagrams to guide instruction. Data collected included maps/diagrams presented in workshops and final maps/diagrams included in regular assignments, student reflections, and researcher's field notes. This paper presents the case study of one student teacher's work (Robert Brennan) on using concept maps to plan a teaching sequence on the topic "Derivatives." Specifically, student teachers were required in Assignment 1, to prepare a teaching sequence on the topic "Derivatives" based on the syllabus notes: *Section 8. The Tangent and the Derivative of a Function* for the *Higher School Certificate (HSC) Mathematics 2/3 Unit -Years 11-12* (NSW BOS, 2002, pg. 50-53). The next sections describe Robert's teaching sequence concept maps.

3 Data Analysis

3.1 Learning to Concept Map

The key characteristics of concept maps, namely the (i) hierarchical organization of key and subsidiary concepts and (ii) inclusion of linking words on connecting lines to form propositions from chains of "node $\xrightarrow{\text{linking words}}$ node" triads, were illustrated and demonstrated through a number of pre-prepared concept maps. During group/individual work in weekly workshops, student teachers practiced concept mapping selected topics/problems/activities and concerns were addressed as they emerged during these activities. For Robert, he identified, selected and ranked key and subsidiary concepts of the selected topic/problem/activity before organising them hierarchically from most general to more specific concepts. While constructing a hierarchy of nodes, reflecting upon the emerging network of interconnections, selecting linking words, critically evaluating and assessing the map's overall validity in terms of the discipline knowledge, Robert inevitably realized that he was thinking more deeply and intensively about possible variations of underlying conceptual structures and cognitively deliberating between alternatives. Not surprisingly, Robert called this preparatory stage "*the verb-type*" by which he meant "*the act of doing the map*" and is *"represented by a pseudo-algorithm to draw the concept map – such as choosing key concepts and possible links*." Explaining his experiences, he said: "*There are in fact 2 'knowledge constructs' gained from doing concept maps. Firstly the 'verb-type', by which I mean the act of doing the map, even if it ends up in the bin at the end. And secondly, the 'noun-type' by which I mean the end product, the actual map of the conceptual structure*." It seems that whilst learning to concept map, Robert realized for himself that "*there are actually 2 types of maps … there is the pre-existing one that is embedded in the mapper's brain, and then there is the map that actually best describes the (unit/problem/activity) for the mapper*." Basically conjecturing that these "*2 maps and their differences could be described by Vygotsky's zone of proximal development*," he explained that, "*I was confused as to whether I was mapping my 'prior knowledge construct map' or the 'map of best description' and so I struggled with the concept maps*." These distinctions (or confusions) between the likely nature and focus of maps were perceived and defined by Robert as "*dimensions*" of a concept map; see Table 1 for his schematic representations of dimensions.

	*Prior-Knowledge Construct Map	#Best Description Map
Verb-type Knowledge Construction	(1) Represented by pseudo-algorithm to draw concept map – such as choosing key concepts & possible links.	(2) Represented by a plan to re-arrange the prior-knowledge construct map to best solve current problem.
Noun-type Knowledge Construction	(2) Final copy of concept map that accurately represents "what is in mapper's head."	(3) Final copy of concept map, which may represent a solution to a mathematics problem or a teacher's unit/lesson plan.

* Already existing and may be primitive or erudite but exists and must be discovered.
\# Varies depending on the nature of the problem; i.e., is it a mathematics problem to be solved; or a content summary of a topic of study by a teacher?

Table 1: Student's Perceptions of "Dimensions" of a Concept Map

Elaborating further, Robert proposed yet another dimension namely the "*focus*" of the map. He wrote: "*By this term I mean the 'qualitative nature' of the map – Is it concrete in nature so that it's usefulness lies in teaching a student to solve a particular mathematics problem; or is it descriptive in nature providing an abstract summary of a topic?*" However, sharing and discussing these reflections later on in-class clarified further for Robert the need to explicate the intended purpose and specific focus of a map, often a common point of confusion when learning to concept map for the first time. That is, explicating the purpose and focus of map first ensures the appropriate selection of concepts, hierarchical organization and suitable linking of relevant nodes to enhance the map's overall cohesiveness and meaningfulness. For example, a concept map of a mathematics problem (Type 1) illustrates the conceptual structure embodied by the problem and underpinning its solution whilst a topic concept map (Type 2) illustrates the conceptual and epistemological structure of the key ideas (i.e. mathematics concepts and principles) relevant to the topic. As a consequence of such qualitative distinctions, Type 1 map would be more contextualized and situated in contrast to the more general overview and abstract Type 2 ones.

Apparently from Robert's reflections (in *italics* above) and schematic representations in Table 1, preparing and constructing a concept map demanded much deliberation and decision-making, cognitive and analytical processing beyond the mere recall of formal definitions and general formulas. As a result, by the time the first assignment was due, Robert had become increasingly more proficient in selecting key and subsidiary concepts with strengthened skills in hierarchically organizing concepts into cohesive groups and more confident in constructing viable networks of propositional links to communicate his understanding of the task's conceptual structure. Explaining this growth in understanding, Robert wrote: "*I realized that these 2 types of maps ['prior knowledge construct map' and 'map of best description'], need to be well-defined before mapping begins*". Through his 2-dimensional schema in Table 1, Robert posed two viable pathways for the construction of a '*best description map*.' Firstly, by progressing vertically down the 'prior-knowledge construct map' (Column 1 of Table 1) from (1) a pseudo-algorithm through (2) a final copy of what is in the mapper's head and then horizontally across to (3) a final copy representing a solution to a mathematics problem or teacher's unit/lesson plan. Secondly, by progressing horizontally along the 'verb-type knowledge construction' (Row 1 of Table 1) from (1) a pseudo-algorithm on to (2) a plan to re-arrange the prior-knowledge construct map and down to (3) a final copy of concept map to represent a solution to a mathematics problem or teacher's unit/lesson plan. The choice of pathways appears dependent on whether the focus is a problem or a unit/lesson. Irrespective of the pathway taken, each seems to represent a progressive or developmental trajectory from an initial preliminary version to a finalized '*best description map*.' Presented below is Robert's final 'best description' maps obtained through the first pathway for the purpose of illustrating a teaching sequence.

3.2 Overview Concept Maps - Mathematics 2/3 Unit-Years 11-12

Instead of designing a teaching sequence directly from syllabus notes, Robert Brennan first of all, situated the topic of "Derivatives" amongst those required for Years 11-12 in the *Mathematics 2/3 Unit* (corresponding to the HSC Mathematics and Mathematics Extension 1 courses, NSW BOS (2002)) to provide a better overview of topics to be taught prior to introducing "Derivatives." Proceeding by identifying the main ideas from syllabus notes, Robert went beyond the requirements of the assignment and constructed 14 overview concept maps (only *two* shown here), which covered a range of Years 11-12 prescribed topics. He commented that: "*For me it seems that I must firstly define the entire space of the (unit) before attempting to define the (unit) itself.*" Shown in Figures 1 and 2 are Robert's first two overview concept maps illustrating some of his organisational hierarchies to depict differentiating levels of generality (Level #) from the most general concepts to progressively more specific concepts towards the bottom of map. For example, Figure 1 is an overview of Year 11 Mathematics (Level 1) that is subsumed under 3 main concepts namely "(A) Building Blocks of Functions", "(B) Real Functions", and "(C) Examples of Functions" at Level 2, with the order A, B, and C indicating a preferred teaching sequence. Relevant to the topic "Derivatives" is the middle branch subsumed under the Level 2 node: "B. Real Functions" with a triple-branching link connecting to 3 less general concepts (at Level 3) namely "I. Foundations", "II. The Slope Problem" and "III. Introduction: Product, Quotient & Chain Rule". Again, the ordering I, II, and III suggests that (I) is the required prior knowledge to the topic "Derivatives" embodied by the middle "II. The Slope Problem" sub-branch (marked *). Similarly, the adjacent "(A) Building Blocks of Functions" branch on the left and the adjacent "(C) Examples of Functions" branch to the right, could be likewise read from top to bottom.

In comparison to Figure 1, Figure 2 on Year 12 Mathematics (Level 1) shows the relevant information in relation to the topic: "Derivatives" such as nodes subsumed under the Level 4 nodes: "HSC (2 Unit) Mathematics topics/units” and “HSC (3 Unit) Mathematics Extension I topics/units” namely “II. Calculus” (marked *) and “I. Applications of Calculus to the Physical World” (marked **). Reading from top-to-bottom, the relevant proposition P1 is: “HSC (2 Unit) Mathematics topics/units *comprises 3 “sub”-strands:* I. Coordinate Geometry, II. Calculus and III. Transcendental Functions *and one unit:* 1. Kinematics”.

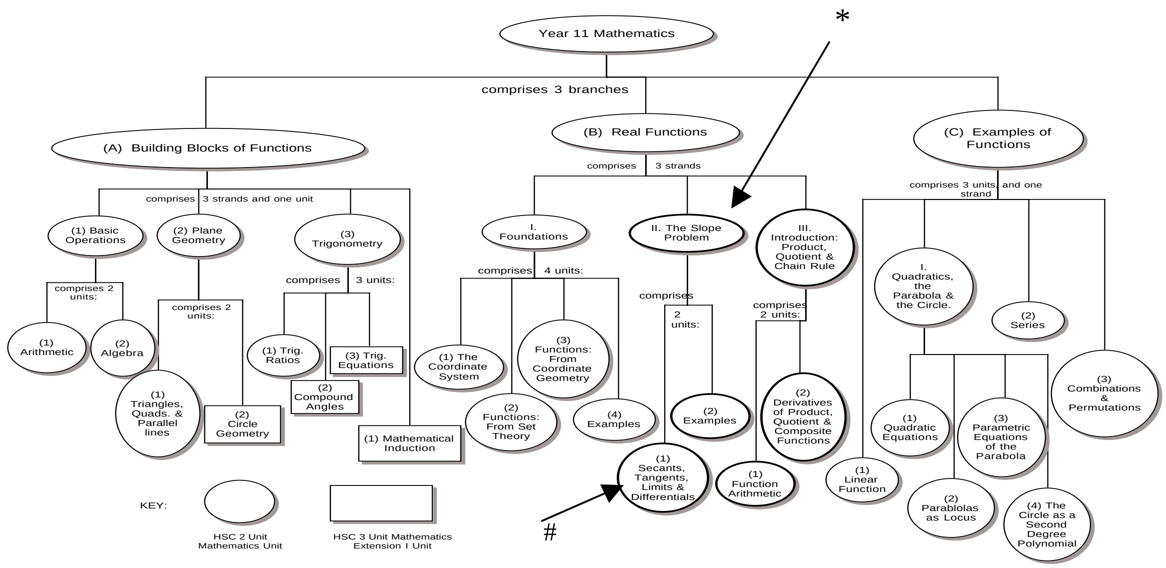

Figure 1. Year 11 overview concept map.

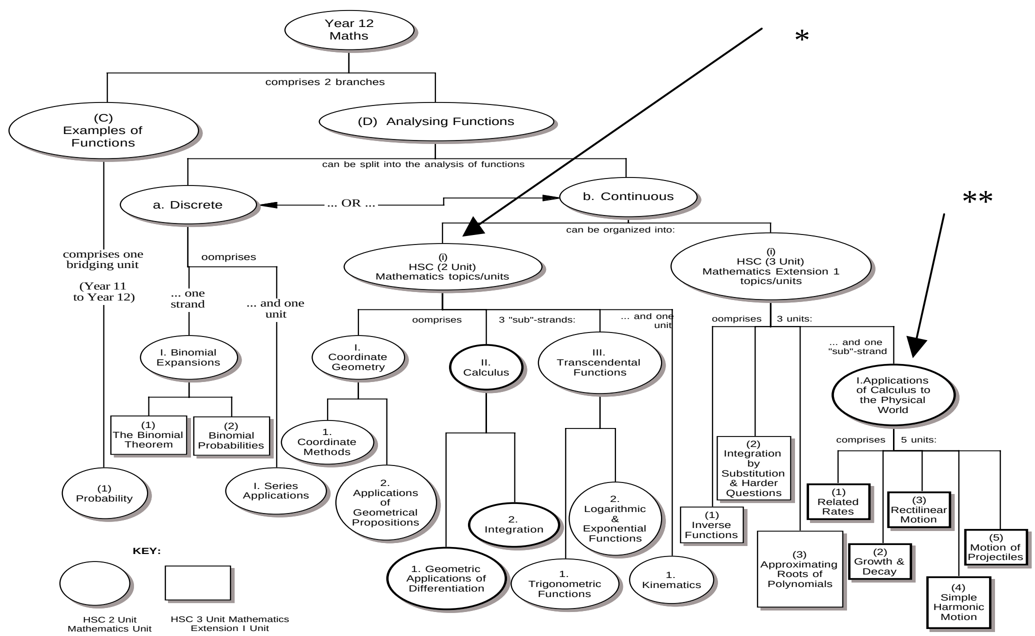

Figure 2. Year 12 overview concept map.

From the “II. Calculus” node is a progressive differentiation double-link to connect to the two terminal nodes “1. Geometric Applications of Differentiation” and “2. Integration” but with no linking words. Situated within the other calculus-related sub-branch (marked **) subsumed under the node: “HSC (3 Unit) Mathematics Extension I topics/units” is the proposition (P2): “1. Applications of Calculus to the Physical World *comprises 5 units:* (1) Related Rates, (2) Growth & Decay, (3) Rectilinear Motion, (4) Simple Harmonic Motion, and (5) Motion of Projectiles”. In fact, Figure 2 clearly depicts the nested structure of HSC 2 Unit Mathematics topics within HSC 3 Unit Mathematics and showing that HSC 3 Unit extends topics initially encountered in HSC 2 Unit Mathematics. This inter-relationship is schematically shown by the left-to-right order of the Level 6 concept hierarchies in Figure

2. Collectively reading from the two maps, Figure 1's middle branch, from left to right illustrates the syllabus' expectation and Robert's plan that the topic "Derivatives" would be introduced via "II. The Slope Problem" (marked *) through secants, tangents, limits and differentials (marked #). In comparison, Figure 2 provides a more general overview of this sequencing of topics but situated within Year 12 HSC 2 Unit Mathematics (i.e. "1. Coordinate Geometry" to be covered prior to "II. Calculus") and including clear distinctions of topics covered as applications of calculus to the real world within HSC (3 Unit) Mathematics (marked **). Following on from this general overview of Years 11-12 Mathematics courses, Robert developed a detailed concept map to illustrate a more developmental approach to "Derivatives" which explicitly builds upon students' prior knowledge of gradients of linear graphs by elaborating further the meaning of the terminal node: "Secants, Limits, Tangents & Derivatives" of Figure 1 (marked #). This process is briefly described next.

3.3 Teaching Sequence Concept Map – Topic "Derivatives"

Robert's critical and conceptual analysis of *Section 8: The Tangent and the Derivative of a Function* (NSW BOS, 2002, pg. 50-53) yielded 19 main groups of sub-topics of which 5 was identified to be the most relevant for introducing derivatives; see Figure 3 for the 5 syllabus referenced sub-topics.

Section 8.3 Gradient of a secant to the curve y = f(x).
Section 8.4a Tangent as the limiting position of a secant.
Section 8.4b The gradient of the tangent.
Section 8.5a Formal definition of the gradient of y = f(x) at the point where x = c.
Section 8.6a The gradient or derivative as a function.

Figure 3. List of sub-topics relevant to "Derivatives" (NSW BOS, 2002).

These sub-topics in Figure 3 eventually formed the basis of Robert's topic concept map for the introduction of derivatives shown in Figure 4. Selecting the node: "Secants, Limits, Tangents and Derivatives" (from Figure 1, marked #) as the titular node at Level 1 of Figure 4, the next hierarchical level showed progressive differentiating triple-links to three main concepts: "(1) The 2-Point Method", "(2) The Limiting Process" and "(3) Derivative Functions" at Level 2. Furthermore, the resulting 3 branches and concept hierarchies appeared organized around the three types of knowledge namely (i) prior knowledge, (ii) new knowledge (i.e. derivatives) and (iii) extensions, reflective of the philosophy of preparing learning activities promoted by the mathematics education unit Robert was enrolled in. Specifically, the leftmost branch indicated the prior knowledge ("(1) The 2-Point Method" branch) described in the syllabus students require before being introduced to the derivative concept. Emanating from the "(1) The 2-Point Method" node is a split-link that generated propositions: (P3): "(1) The 2-Point Method *for finding the gradient of* a) straight lines" and (P4): "The 2-Point Method *for finding the gradient of* b) Secants (8.3)" where 8.3 was a reference to syllabus notes, Section 8.3 (NSW BOS, 2002, pg. 50) and the first of the 5 sub-topics listed in Figure 3. From the middle Level 2 node: "(2) The Limiting Process" are two progressive differentiating split-links to Level 3 nodes: "Geographically" and "Algebraically" which form the extended proposition P5: "(2) The Limiting Process *which can be looked at* Geographically *by introducing* the 3 types of points *which are* Fixed: P(c, f(c)), Moveable: Q(u, f(u)), and General R(x, f(x))". Emanating from the "Moveable: Q(u, f(u))" node is a split-link that form propositions P6: "Moveable: Q(u, f(u)) *can generate tangent at point P(c, f(c)) by:* a) Moving Q to P (8.4a)" and P7: "Moveable: Q(u, f(u)) *can generate tangent for any point on curve R(x, f(x)) by:* b) Moving Q to R (8.4a)". On the other hand at the Level 3 node: "Algebraically" of the middle branch, are two differentiating links which formulated an extended proposition P8: "The Limiting Process *which can be looked at* Algebraically *by noting the change of* x: Δx=c–u and *by noting the change of* y: Δy=f(c)–f(u)". The subsequent merging of cross-links (i.e., integrative reconciliation) from the two Level 4 nodes "Δx=c–u" and "Δy=f(*c*)–f(u)" formulated proposition P9: "Δx=c–u, Δy=f(*c*)–f(u) *which yields* $m=\frac{\Delta y}{\Delta x}$" with a single link to the Level 6 node to form the extended proposition P10: "$m=\frac{\Delta y}{\Delta x}$ *and the language for moving Q to P is*: $\lim_{u \to c} \left(\frac{f(c)-f(u)}{c-u}\right)$ *which gives:* The gradient of the tangent at x = c. (8.4b) *which is:* a) Denoted f'(*c*), and called the differential coefficient of f(x) at c. (8.5a)" The middle branch evidently focuses on the geometric introduction of a tangent and the algebraic representation of the limiting gradient as a differential coefficient. In contrast to the middle branch, the rightmost branch depicts the progressive development (or extension) of the concept "differential coefficient f'(c)" (marked *) to the more general concept "Derivative Functions" (Level 2). Specifically, the first proposition (P11) is: "(3) Derivative Functions *are best studied by introducing the identity*: *u*=x+Δx" followed by the triple-pronged proposition (P12) "*u*=x+Δx *which yields:* f(u)=f(x+Δx), u–x=Δx, u→x ≡ Δx→0" (Level 4). Cross links from the latter nodes merged to form the proposition P13: “f(u)=f(x+Δx), u–x=Δx, u→x ≡ Δx→0 *which upon substitution give* $\lim_{u \to x} \frac{f(u)-f(x)}{u-x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$” *which is:* “a) Denoted f’(x), and called the derivative function of f(x). (8.6a).”

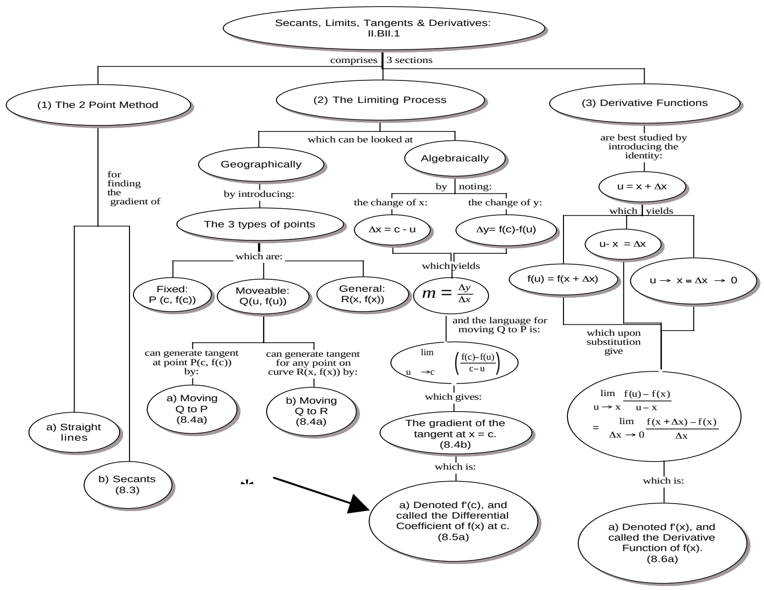

Figure 4. Topic “Formal definition of derivatives” concept map

Overall, Figure 4 shows a topic concept map with an explicit organization into 3 main branches, which implicitly suggests a teaching sequence from left to right. Furthermore, within each concept hierarchy, there is a logical development of ideas implied by reading from the top to the bottom levels and from left-to-right. Similarly, when reading from the terminal node of a (sub-)branch up to the top level of the adjacent concept hierarchy to the right as described above. The advantage of the visual and more informative display of the interconnectedness of key ideas with respect to each sub-topic (i.e. 8.4a, 8.4b and 8.5a) is clearly depicted by comparing each of the three sub-branches subsumed under the “(2) The Limiting Process” middle branch to the linear sequential list in Figure 3. Of additional interest is the explicit connections between concept maps such as the link between the titular node: “Secants, Limits, Tangents and Derivatives” of Figure 4 and the same-named node in Figure 1 (marked #). Taken together, Figures 1, 2 and 4 clearly illustrate a visual trend from the macro view of main topics in a 2-year mathematics curriculum (Figures 1 and 2) to a micro-view of key and subsidiary concepts within a sub-topic (Figure 4); that is, there is an apparent increasingly more detailed elaboration of conceptual interconnections most relevant to “Derivatives” when moving from Figures 2, 1 to 4.

4 Discussion

The discussion of findings are organized around four main points namely (1) concept maps of critical and conceptual analysis, (2) workshop discourse, (3) socio-mathematical norms, and (4) practical management of the learning ecology within weekly workshops. Each issue is briefly discussed next.

Concept Maps of Critical Analysis - Robert’s overview concept maps in Figures 1 and 2 provided a big picture view of the Years 11 and 12 topics within which the topic “Derivatives” is situated. Each map represented what Robert had categorized as “*final copy*” of a concept map to accurately represent a teacher’s unit plan. The visual positioning of concepts within hierarchies and the overall grouping of relevant hierarchies, not only suggested potential teaching sequences when the map is read from left-to-right, but it also depicted the level of generality of ideas and/or concepts when read from top-to-bottom. Together, they defined a unique position for a node/hierarchy, roughly paralleling that of a point on the Cartesian plane, whilst simultaneously denoting a relative position amongst a network of nodes/hierarchies highlighting the interrelatedness of ideas. Findings demonstrated that concept maps provided a parsimonious, visual organization of interconnecting ideas, not only at the macro-level (Figures 1 and 2), but also at a relatively more in-depth micro-level of a teaching sequence (Figure 4), which collectively enriched the design of a teaching sequence. The cognitive processes of identifying key and subsidiary concepts, hierarchically organising them, constructing and finalising concept maps necessarily required that the student teacher reflected deeply upon his own knowledge of mathematical concepts and processes whilst determining the most viable, visual hierarchical organizations of interconnections he anticipated would promote his future students' conceptual understanding of derivatives. Labelling the preparatory version his "*verb-type*" or "*prior knowledge*" map, he proposed that this was a necessary step before finalizing a "*noun-type*" or "*map of best descriptions*." His reflective practice when mapping subsequently led him to develop a two-dimensional schema of "*verb-type→noun-type*" by "*prior-knowledge→best-description*" to illustrate qualitative differences between types of maps. Although cognitive demands on the student teacher to critically analyse syllabus documentation, whether or not a concept map is used prior to developing a teaching sequence would probably be very similar, the significant difference however, is in the extra cognitive and meta-cognitive skills to meaningfully and visually organise ideas into hierarchies of propositional links to display and highlight the "interconnectedness" of concepts across different levels of generality and specificity. Hierarchically organizing concepts evidently challenged Robert to clarify his thinking as he sought out mathematical principles to provide underlying frameworks that enhance the cohesiveness and meaningfulness of nested hierarchies (branch). This cognitive exercise appeared to demand reflective, lateral and deeper thinking about mathematics concepts and processes in order to construct visual and schematic representations of meaningful and cohesive knowledge systems (e.g. Figures 1, 2, and 4) in contrast to a sequential and linear view of topics from reading notes (e.g. Figures 3).

Workshop Discourse - The kinds of discourse that emerged during critiques of presented maps in workshops involved interactions and exchanges of ideas between the student teacher presenting his/her own map and his/her peers and lecturer-researcher responding and making critical comments usually in the form of requests for clarifications, recommendations for additions/deletions, or confirmations of presented information. Consequently over the semester, Robert learnt to interact and respond appropriately to critical comments as he argued the correctness of his maps, provided counter-arguments to points raised by his peers, or sought modifications of maps when justifiable. Through these social negotiations, argument and debate, the student teacher demonstrated growing awareness of the importance of adjusting the level of his mathematical language (manifested as concept labels and linking words) to be consistent with the recommended level of the syllabus' staged outcomes. Furthermore, students voluntarily shared their reflections of their experiences simultaneously encouraging others to do the same. Ensuing discussions therefore, focussed on how their mapping experiences impacted on the way they planned, thought and viewed the development of teaching sequences/learning activities. For example, Robert discussed initial difficulties as he learnt to concept map problems/activities/units such as the difficulty of identifying appropriate and concise labels for main ideas, clarifying the purpose and focus of maps, and determining the most suitable hierarchies. However, through workshop discourse, Robert's concerns were eventually clarified. Through the discussion of his reflections, he demonstrated an in-depth engagement and reflective practice with the task of concept mapping which previously and independently prompted him to schematise the mapping process as "*dimensions*" to qualitatively clarify the purpose and focus of concept maps.

Socio-mathematical norms - The types of participation norms established in workshops included participation in group/class analysis of key and subsidiary ideas in topics/problems/activities; the transformation of analysis results into concept maps leading to group/class co-construction of exemplar maps; class critiques of individually constructed maps; and discussions of student reflections and mapping experiences. Finally, established socio-mathematical norms influenced, modulated and directed the dynamics of group/class discussions and critiques in weekly workshops. Undoubtedly, these norms impacted the way Robert planned and developed his final 'best description maps' of a teaching sequence as presented here.

Practical Management of the Learning Ecology - The types of practical means by which the lecturer-researcher "orchestrated relations among [the different] elements" (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003, p. 9) included selecting appropriate tasks (activities/problems/topics) to introduce concept mapping, providing support to students whilst they were learning for the first time, critiquing their work and setting more tasks to challenge their critical abilities and skills not only of concept mapping but including critical analysis of syllabus outcomes. The lecturer-researcher also facilitated group discussions and critiques during map presentations, and coordinated the sharing of students' reflections as materials for discussion of the impact of concept mapping on their own 'thinking about learning' and 'thinking about teaching.' With workshop presentations and reflection sessions focussing on concept maps, ensuing discourse brainstormed multiple ways in which classroom activities could be supported and facilitated through having their future students present and communicate their mathematical understanding via concept maps. Whilst the actual involvement of school students in concept mapping was not part of the main study, using concept maps by student teachers as pedagogical tools was.

5 Main Insights

With the acquired expertise and proficiency in constructing concept maps, the student teacher was empowered to use these tools innovatively (i) to critically analyse syllabus outcomes, and (ii) to design a suitable teaching sequence by hierarchically and visually clarifying prior knowledge and future knowledge and using appropriate mathematical language to effectively communicate staged-appropriate mathematics content. Since completed, practice and final maps encapsulated both the conceptual and epistemological frameworks of a topic, through their construction, the student teacher routinely searched for connections between key and subsidiary concepts, and whilst doing so, he made insightful observations about the qualitative distinction between the nature of maps, depending on their purpose and focus, in terms of a two-dimensional schema, to distinguish between maps that are more abstract as in topic concept maps or those that are more concrete as in problem concept maps. For example, he also distinguished between dimensions of a concept map when used as a metacognitive tool to collect his thoughts and ideas about the focus of the map (verb-type) and a final concept map described as his 'best-description map (noun-type). A significant advantage of being proficient in concept mapping is the acquisition of critical skills that can be usefully applied to many situations such as demonstrated through his additional effort to situate the assigned topic within the macro picture of the two-year mathematics curriculum.

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7 References

Afamasaga-Fuata'i, K., (2005). Students' conceptual understanding and critical thinking? A case for concept maps and vee diagrams in mathematics problem solving. In M. Coupland, J, Anderson & T. Spencer (eds). *Making Mathematics Vital*. Proceedings of the Twentieth Biennial Conference of the Australian Association of Mathematics Teachers (AAMT), (pp. 43-52). January 17 – 21, 2005. University of Technology, Sydney, Australia.

Afamasaga-Fuata'i, K. (2004a). Concept maps and vee diagrams as tools for learning new mathematics topics. In A. J. Canãs, J. D. Novak & Gonázales (eds). *Concept Maps: Theory, Methodology, Technology*. Proceedings of the First International Conference on Concept Mapping September 14-17, 2004 (pp. 13 – 20). Dirección de Publicaciones de la Universidad Pública de Navarra, Spain.

Afamasaga-Fuata'i, K. (2004b). An undergraduate's understanding of differential equations through concept maps and vee diagrams. In A. J. Canãs, J. D. Novak & Gonázales (eds). *Concept Maps: Theory, Methodology, Technology*. Proceedings of the First International Conference on Concept Mapping September 14-17, 2004 (pp. 21 – 29). Dirección de Publicaciones de la Universidad Pública de Navarra, Spain.

Ausubel, D. P. (2000). *The Acquisition and Retention of Knowledge: A Cognitive View*. Dordrecht; Boston: Kluwer Academic Publishers.

Bobis, J., Mulligan, J., & Lowrie, T. (2004). *Mathematics for Children. Challenging children to think mathematically.* Pearson Prentice Hill, Australia.

Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher*, 32(1):9-13.

New South Wales Board of Studies (NSW BOS). (2002). Mathematics K-6 Syllabus 2002.

Novak, J. D. (2004). A science education research program that led to the development of the concept mapping tool and new model for education. In A. J. Canãs, J. D. Novak & Gonázales (eds). *Concept Maps: Theory, Methodology, Technology*. Proceedings of the First International Conference on Concept Mapping September 14-17, 2004 (pp. 457 – 467). Dirección de Publicaciones de la Universidad Pública de Navarra, Spain.