ONLINE MATHEMATICS WITH INTERACTIVE CONCEPT MAPS

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Abstract. There has not been a free and easy-to-use utility to handle both authoring and presentation of large graphs online, especially with mathematical content. On one hand, concept map editors are ideal for authoring graphs, but publishing capabilities may be limited. On the other hand, numerous browsing and layout tools exist for publishing content generated elsewhere. We present a transformation tool Xcm2kg that tries to solve the problem by integrating authoring with IHMC CmapTools to publishing based on TouchGraph. GXL is used as an intermediate format to facilitate interoperability with other graph-based applications. The tool is demonstrated with concept maps drawn on an undergraduate mathematics course.

1   Introduction: Concept Mapping for Mathematics

Concept maps enable representing large thematic entities in compact form. Concept maps are effective as constructive learning tools (Novak, 1990), and they have been successfully applied to multitude of domains. Although there have been several studies related to concept mapping in mathematics (Afamasaga-Fuata'i, 2004; Baralos, 2002; Caldwell et al., 2006; Fuhrmann, 1999; Kujansuu, 2003; Schmittau, 2004), we believe that concept mapping has yet more to offer for both mathematics education and general description of mathematical content. Mathematics is well suited for hypertext representation (Mayans, 2004), and concept maps can be interpreted as localized visualizations of hypertext. Mathematics is grounded on precise concepts that are further based on more elementary concepts, formally linked with *definition-proposition-proof* –chains. Concept maps allow one to see the main concepts and results of a theory at a glance, helping to follow the proofs as well.

To our knowledge, there is no free and usable utility to handle both authoring and presentation of large graphs with mathematical content online. The problem lies in the inability of the current concept mapping software to present conveniently anything else than textual data, and the lack of interoperability between them. Recent programs usually use XML-based storage formats, but no *de facto* format does exist. Mathematical formulas may have to be converted manually to images, which is not practical with large texts. MathML (MathML, 2003) helps, but needs software support both for editing and publishing. TeX-like concise formatting, supported by some publishing packages (e.g. PhpMathPublisher, http://www.xm1math.net) and wikis (e.g. MediaWiki, http://www.mediawiki.org) would be more efficient, especially for mathematically oriented users.

In this paper, we present a transformation tool Xcm2kg that tries to solve this authoring vs. browsing problem by supporting concept map authoring with IHMC CmapTools (http://cmap.ihmc.us) and publishing with enhanced TouchGraph (http://touchgraph.sf.net) technology. Xcm2kg also facilitates interoperability with other graph drawing applications by using GXL (Graph eXchange Language) as an intermediate format. GXL is a data interchange format between graph-based presentation and software analysis tools (Winter, 2001). We demonstrate the tool by converting hand-drawn concept maps from an undergraduate mathematics course (Kujansuu, 2003) to CmapTools and comparing them with TouchGraph-based user interface KeyGraph.

2   From Concept Maps to Interactive Graphs

In this section, we describe Xcm2kg transformation tool with related software.

2.1   IHMC CmapTools

IHMC CmapTools (Cañas et al., 2004) is a Java-based concept mapping environment developed by the Institute of Human and Machine Cognition. Its strength lies in ease of use and modeling graphical layout of nodes and edges. Model elements can also contain additional resources like pictures, other documents, concept maps, and WWW links. Concept maps are modeled as hypergraphs: edge can be connected to multiple nodes. Mathematical content can be written using a WYSIWYG MathML editor. CmapTools supports several output formats: concept maps can be exported as a textual outline, HTML document, or in multiple XML formats.

While CmapTools is adequate for authoring, its default HTML output has limitations, because concept map is exported as a single image. This can be impractical with large (over 50 nodes) graphs – especially if the graph is not partitioned to subgraphs. A limitation in graph metamodel is that unlike in UML, roles cannot be attached to endpoints of an edge. User can still choose whether the edges are directed or not. Despite its minor

shortcomings, CmapTools is suitable for creation of reduced size concept maps. Using a distributed server, the software also works as collaborative tool, where maps can be distributed and even edited simultaneously.

2.2 TouchGraph LinkBrowser

TouchGraph is an innovative, partially open source graph browsing component developed by Alex Shapiro. TouchGraph supports interactive browsing of arbitrarily large graphs because the view can be restricted to a local portion of the graph. The graph is browsed in an applet. Links can be opened in a separate frame. Despite of the visually attractive layout, the user can get lost in a large graph, which moving of the nodes does not help. Naturally, careful design of the graph structure and appearance of the nodes makes the navigation easier. Edges in the graph are directed, but no additional attributes, e.g. linking phrases, can be attached to them.

TouchGraph LinkBrowser is an application developed from the user interface component, which attaches a URL link to each node, making it possible to use LinkBrowser to navigate WWW pages. Additionally, each node can have a *tooltip* (hint) which is shown if the mouse cursor is left on the node. The hint can show HTML-formatted text, including pictures and URL-links. TouchGraph LinkBrowser uses an XML-based format for graph presentation. LinkBrowser can also be used as a basic graph editor, but creating graphs is rather tedious compared to concept mapping software, especially if nodes link to external resources.

2.3 Xcm2kg and KeyGraph

Xcm2kg is a conversion utility that converts concept maps from CmapTools XCM format to KeyGraph, a graph visualization component based on TouchGraph. The conversion is done with two separate filters, Xcm2gxl and Gxl2TouchGraph. The overall process is shown in **Figure 1**. External resources (i.e. pictures, URL links, concept maps) are shown as hints in LinkBrowser nodes. References to external concept maps are passed to a PHP script that loads the concept map into an applet. MathML markup is converted to images and shown directly in TouchGraph nodes similar to background pictures. Xcm2kg is implemented in Java and published as open source as a part of ConceptUtils transformation framework (http://conceptutils.sf.net).
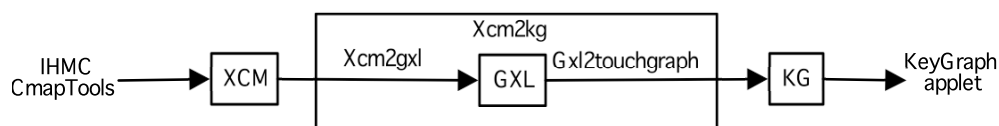
Figure 1. Xcm2kg transformation tool chain.

Xcm2gxl converts XCM file from CmapTools to a generic graph interchange format, GXL. JGraph (http://www.jgraph.com) component is used for internal representation of the graph model during conversion. MathML markup is converted to images using JEuclid (http://jeuclid.sf.net). References and node/edge labels are preserved as attributes. Layout information is presented as a *style* attribute. Using GXL in transformation enables the conversion of any GXL-formatted file to TouchGraph readable format. For example, JGraphpad editor in JGraph package supports GXL, and Graphviz toolkit (http://www.graphviz.org) contains bidirectional DOT-GXL transformation utilities. Representation of attribute data can differ between GXL files acquired from different sources and layout is generally not preserved, but at least the structure of the graph is representable.

Gxl2TouchGraph converts GXL file to KeyGraph format. KeyGraph is an extension to TouchGraph LinkBrowser 1.20, improving the user interface and expanding the graph format. Enhancements to XML format include edge labels (linking phrases), support for background images, and multiline text. Keyboard-based navigation was added to user interface. If multiple nodes are present, the one nearest to the arrow direction is selected. Pictures can be shown as hints or directly in LinkBrowser nodes. URL links can be local (opened in another frame) or external (opened in new browser window). N-ary relations supported by CmapTools are resolved in KeyGraph by generating corresponding edges for all possible combinations. Finally, KeyGraph includes a simple framework that eases publishing concept maps and linking between maps. A PHP-script generates a list of available KeyGraph files and frames for the applet, concept map lists, and local links.

3   Transformation Example

The transformation example is based on an undergraduate mathematics course (Kujansuu, 2003). **Figure 2** is one of the student-drawn maps, which were evaluated and compared to another map drawn by the teacher.
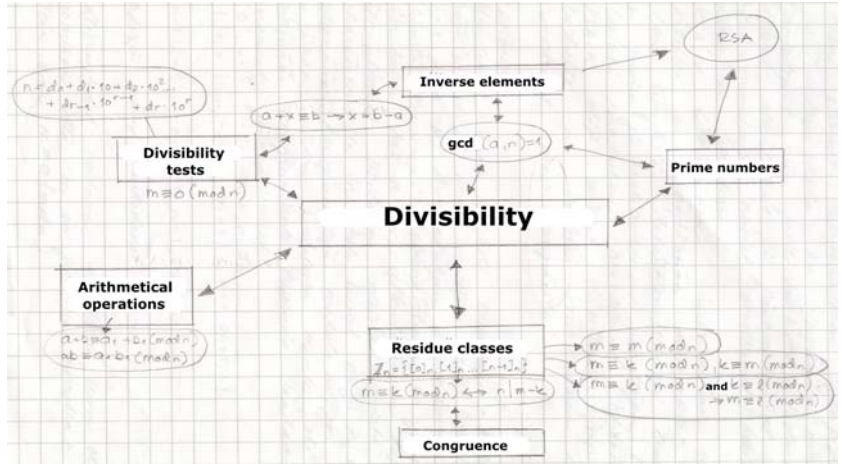
Figure 2. Original, hand-drawn concept map (labels translated from Finnish).

Figure 3 shows the HTML page generated by CmapTools with alternative ways of encoding mathematical text: the image in *Residue classes* was generated using TeX. The formulas under *Congruence* and beside *Series representation* were written using the formula editor, represented as Unicode text and MathML respectively.
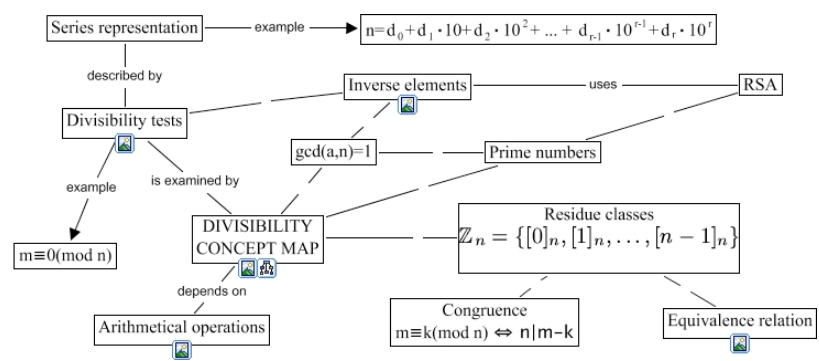

Figure 3. Web page concept map generated by IHMC CmapTools.

Figure 4 shows the dynamic view generated by Xcm2kg in KeyGraph. Note the slight differences in formatting generated from different sources. The hint in *Equivalence relation* links to TeX-generated images.
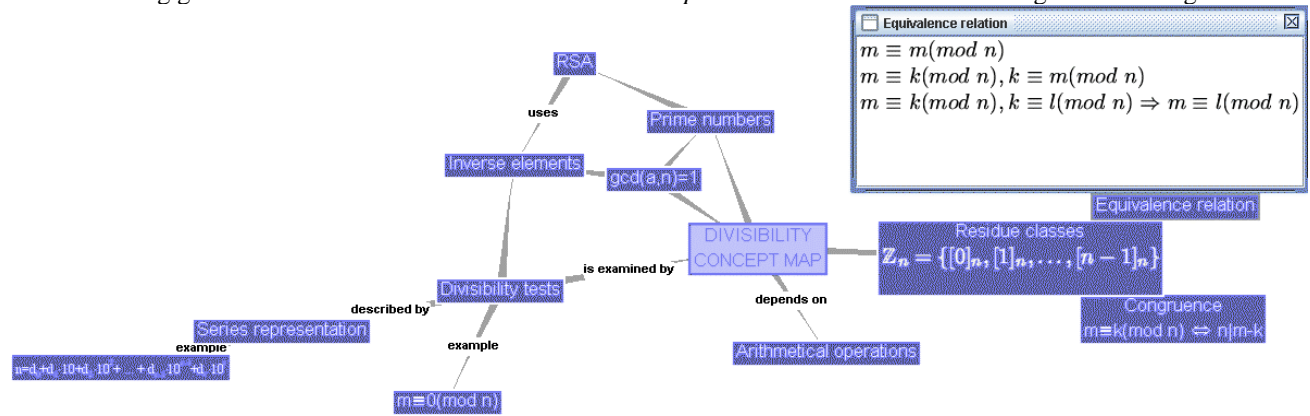

Figure 4. Concept map in KeyGraph visualization.

Even with a small map like this one can see the benefits of the visualization. The view is dynamic and interactive. Resources in tooltips are shown with less effort compared to HTML generated by CmapTools. The visible locality length and zoom can be configured, adapting the view to graphs and nodes of different size.

4   Discussion

Xcm2kg is currently a prototype. Xcm2kg does not preserve font, color or other style information about the graph except relative node positions. All such info is stored in the XCM file, so basically some of it could be

used in conversion as well. Another problem is related to resource file names in XCM output. When concept map is exported to XCM, resource file names are not preserved. The names are converted similar to their description strings that are shown in the concept map. This can lead to name collisions if different resources are described with same names. Fortunately, the problem is not present in CmapTool's newer concept map format CXL, but a transformation component for it is not yet implemented. The output format Xcm2kg uses is downwards compatible with TouchGraph LinkBrowser 1.20. However, Xcm2kg features some extensions that are implemented only in KeyGraph package, the most visible additions being edge labels and image nodes.

Future work in Xcm2kg includes generalized support for GXL-KeyGraph -conversion. This would require a standard way to represent style in GXL files. A promising approach might be using a SVG file for describing layout, separating it from the data model (Minas, 2002). In general, support for visual styles used in CmapTools should be improved. The ConceptUtils framework should be extended to account new formats in transformation, such as FreeMind (http://freemind.sf.net) mind maps. Finally, the tool should be tested more extensively in different settings, especially with mathematical content. For example, concept mapping could be applied to works like *Comprehensive Mathematics for Computer Scientists* (http://math.ifi.unizh.ch/bmwcs), or Metamath (http://metamath.org) that contain cross-referenced indexes about definitions and proofs. Other online mathematics collections, such as Wikibooks (http://en.wikibooks.org/wiki/Mathematics_bookshelf), or Weisstein's *MathWorld* (http://mathworld.wolfram.com) would also benefit from visual representation of the context.

5   Summary

Xcm2kg converts concept maps from IHMC CmapTools XCM format to KeyGraph, a graph user interface format based on TouchGraph. GXL is used as an intermediate format, making the conversion framework extensible. Xcm2kg's purpose is to simplify concept map authoring and web publishing, especially in the mathematics domain – both for learning material and general description of mathematical content. While being a simple converter and a prototype, it integrates CmapTools and KeyGraph in a natural way.

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