Concept Mapping: Connecting Educators
Proc. of the Third Int. Conference on Concept Mapping
A. J. Cañas, P. Reiska, M. Åhlberg & J. D. Novak, Eds.
Tallinn, Estonia & Helsinki, Finland 2008

CONCEPT MAPPING & VEE DIAGRAMMING A PRIMARY MATHEMATICS SUB-TOPIC: *"TIME"*

Karoline Afamasaga-Fuata'I, University of New England, Australia
Email: kafamasa@une.edu.au

A case study investigated a primary preservice teacher's use of concept mapping to interpret 'time' syllabus outcomes of a primary mathematics curriculum and vee diagrams to analyse problems. Findings suggest the preservice teacher developed enhanced skills to critically analyse a topic and a problem. She competently provided mathematical justifications for methods in terms of principles and concepts that are appropriate for primary students, which reflected her developing pedagogical content knowledge and understanding of syllabus outcomes. Constructing concept maps and vee diagrams enabled the preservice teacher to effectively communicate her mathematical ideas, and as a result, developed a deeper, conceptual understanding of the developmental and sequential nature of the mapped sub-strand across the different stages of primary mathematics. Implications for teaching primary mathematics are provided.

1 Introduction

The underlying principles of the New South Wales Board of Studies' *K-6 Mathematics Syllabus* (NSWBOS, 2002) encourage teachers to develop students' conceptual understanding through an appropriate sequence of learning activities and implementation of working mathematically strategies in the classroom. Hence preservice teachers need to develop deep understanding of the mathematics they are expected to teach their future students. To facilitate the development of a primary preservice (PPS) teacher's deep understanding of a syllabus, a case study was conducted to examine the usefulness of the metacognitive tools of hierarchical concept maps (maps) and vee diagrams (diagrams) as means of facilitating conceptual analyses of syllabus outcomes and mathematics problems. For this paper, the focus question is: "*In what ways do hierarchical concept maps and vee diagrams facilitate the development of a primary preservice teacher's deep understanding of the syllabus outcomes of the time sub-strand*?" The Bachelor of Education PPS teacher (i.e., Susan) concept mapped and vee diagrammed over a semester, in her third year mathematics education course in a regional Australian university.

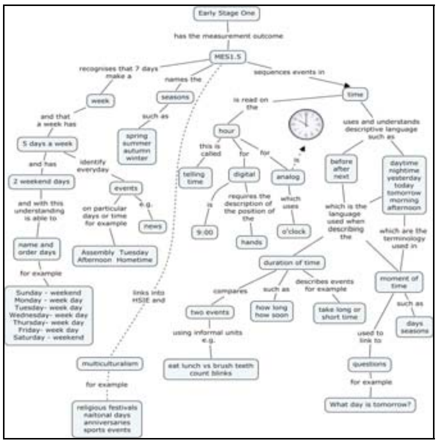
Ausubel's theory of meaningful learning, which defines meaningful learning as learning in which students actively make connections between what they already know and new knowledge, underpins concept mapping particularly its principle that learners' cognitive structures are hierarchically organized with more general, superordinate concepts subsuming less general and more specific concepts. Linking new concepts to existing cognitive structures may occur via *progressive differentiation* (reorganization of existing knowledge under more general ideas) and/or *integrative reconciliation* (synthesising many ideas into one or two when apparent contradictory ideas are reconciled) (Ausubel, 2000; Novak & Cañas, 2006). Others (Jonassen, Peck & Wilson, 1999; Wiske, 1998) also argue that for learning to be meaningful, students must be actively engaged with the learning activity, know their learning goals, construct and reflect on the activity and interpret the results, and self-monitor their progress towards their goals. Further, each discipline has its main concepts and ideas and methods of inquiry. As students seek out interconnections at both the conceptual and procedural levels, they create meaningful relationships and continually reinforce their understanding, which should guide them to solve challenging problems and conduct investigative activities. As an analytical and learning tool, hierarchical concept maps, first introduced by Novak to unpack students' understanding of science concepts based on analyses of interview transcripts, are used to illustrate the hierarchical interconnections between main concepts (nodes) of a knowledge domain with descriptions of the interrelationships (linking words) on the connecting lines. The basic semantic unit (*proposition*) describes a meaningful relationship as shown by the triad "valid-node→valid-linking-words→valid node" (Novak & Gowin, 1984). Vee diagrams, in contrast, were introduced by Gowin as an epistemological tool, in the shape of a vee that is contextualised in the phenomenon to be analysed, as a means of guiding the thinking and reflections involved in making connections between methods of inquiry and the structure of a discipline. The vee's left side depicts the philosophy and theoretical framework driving the analysis to answer the focus question. On the vee's right side are the records, methods of transforming the records to answer the focus question and value claims. The epistemological vee was later modified (Afamasaga-Fuata'i, 2005, 1998) to guide the thinking and reasoning involved in problem solving (an example is presented later). Uses of concept maps and/or vee diagrams as assessment tools of students' conceptual understanding have been examined over time in the sciences (Novak & Cañas, 2004; Mintzes, Wandersee & Novak, 2000) and mathematics (Afamasaga-Fuata'i, 2004, 1998; Hannson, 2005; Williams, 1998). Investigations of the usefulness of maps/diagrams found students' mapped knowledge structure became increasingly complex and integrated as a consequence of multiple iterations of social critiques, revisions and presentations over the semester (Afamasaga-Fuata'i, 2007, 2004). Others also demonstrated the value of maps as pedagogical planning tools to provide topic overviews (Brahier, 2005; Afamasaga-Fuata'i, 2006; Afamasaga-Fuata'i & Reading, 2007), and the usefulness of diagrams to scaffold students' thinking and reasoning and to illustrate interconnections between theory and application in mathematics problem solving (Afamasaga-Fuata'i, 2007, 2005), scientific inquiry (Mintzes, Wandersee & Novak, 2000) and epistemological analysis (Novak & Gowin, 1984).

2 Methodology, & Data Analysis

The case study began with a familiarisation phase, which introduced Susan to the metacognitive strategies using simple topics such as fractions and operations with fractions. Constructing a comprehensive, hierarchical concept map of a mathematics topic selected from the primary mathematics syllabus, and diagrams of related problems to demonstrate the applications of the mapped concepts was the required project of the course. There were three phases to the project. Phase 1 (Assignment 1) required that Susan compile an initial list of concepts, based on a conceptual analysis of the relevant syllabus outcomes, and then to construct an initial topic concept map and diagrams of problems. These were presented and critiqued in class before returning for further revision and expansion. Phase 2 (Assignment 2) involved the presentation of a more structurally complex, expanded concept map and diagrams of more problems. These were socially critiqued and returned for further revision and expansion. Phase 3 (Assignment 3) was the final submission of a more comprehensive, hierarchical topic concept map and more diagrams of related problems and activities, which extended previous work and incorporating comments from previous critiques, and including a journal of reflections. Data collected included maps and diagrams from the familiarization phase, weekly workshops, and three phases of the main project including a journal of reflections. This paper presents samples of Susan's submitted work in Phase 1 to illustrate the application of maps/diagrams as learning, analytical and pedagogical tools for the *Time* sub-strand of the *NSW BOS K-6 Mathematics Syllabus* (NSWBOS, 2002). Her work with the *Area* sub-topic of the measurement content strand is reported in Afamasaga-Fuata'i (2007). Susan's concept maps illustrate her critical analyses and interpretations of the *Time* Knowledge & Skills (K&S) and Working Mathematically (WM) Syllabus Outcomes from *Early Stage One to Stage 4*, and an example vee diagram explicating her critical analysis of a mathematics problem. Concept maps are qualitatively analysed by considering the meaningfulness and interconnectedness of the networks of propositions each displays and qualitatively compared to statements of syllabus outcomes. For this paper, concept maps are read from top to bottom in following a link and across from left to right for branches unless otherwise specified. Excerpts from Susan's reflection journal are also provided to support her concept map and vee diagram data.

3 Syllabus Outcome Concept Maps

MES1.5 syllabus outcomes involve the introduction of the concepts of week, seasons and time as interpreted by Susan and viewed in Figure 1a. The leftmost branch focuses on elaborating a week as 5 days and 2 weekend days with the final node listing the names and type of days (week day or weekend). A progressive differentiating link from the 5 days a week node illustrates the identification of everyday events on particular days or time such as assembly on Tuesday and news. The adjacent branch focuses on naming the four seasons as illustrated. The rightmost branch comprises three sub-branches. The left sub-branch splits further into three smaller sub-branches. The first one describes the process of reading time on the hour as telling time, for digital (second sub-branch), an example is provided, it requires a description of the position of the hands, and for analog (third sub-branch), reading on the hour uses the term o'clock and the analog clock as provided. With the last two sub-branches of the time branch, examples are provided of the language used to describe time on the left (e.g., before, after, next) and the right (e.g., daytime, nighttime, …, afternoon) before integratively merging at the linking words and then progressively differentiating to two nodes, namely, duration of time on the left, and moment of time on the right. With progressive differentiating links from the former node, they illustrate the comparison of the duration of two events by using informal units (on the left) (e.g., eat lunch vs brush teeth), using question phrases (how long, how soon)(middle link) and using descriptive phrases such as take long or short time (right link). From the moment of time node are two progressively differentiating links, one illustrates the use of descriptive language to ask questions (e.g., What day is tomorrow?) and the other exemplifies that moment of time can be described in days or seasons. Also provided is a single link from the MES1.5 node at the top, which describes a connection to the HSIE subject and multiculturalism as illustrated by religious festivals, national days, anniversaries and sports events. HSIE, Human Society and its Environment, is one of the Key Learning Areas in New South Wales, Australian schools. Overall, this stage focuses on describing the duration of events using everyday language, sequencing of events in time, naming days of the week and seasons and telling time on the hour on digital and analog clocks. A comparison to the list of outcomes (Figure 1b) indicates that all of the key ideas and working mathematically strategies have been included as requested.



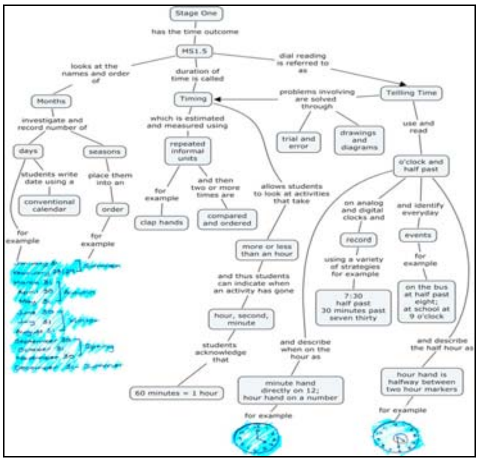
(a)

Early Stage 1 — Time	
MES1.5 Sequences events and uses everyday language to describe the duration of activities	**Key Ideas** Describe the duration of events using everyday language Sequence events in time Name days of the week and seasons Tell time on the hour on digital and analog clocks
Knowledge and Skills	**Working Mathematically**
Students learn about • using and understanding terms such as 'daytime', 'night-time', 'yesterday', 'today', 'tomorrow', 'before', 'after', 'next', 'morning' and 'afternoon' • sequencing events in time • comparing the duration of two events using informal methods eg 'It takes me longer to eat my lunch than it does to clean my teeth.' • recalling that there are seven days in a week • naming and ordering the days of the week • relating events to a particular day or time of day eg 'Assembly is on Tuesday', 'We come to school in the morning.' • naming the seasons • classifying week-days and weekend days • reading hour time on a digital and an analog clock • using the term 'o'clock'	**Students learn to** • ask questions related to time eg 'How long is it until lunchtime?', 'Is tomorrow Wednesday?' (*Questioning*) • describe events that take 'a long time' and events that take 'a short time' (*Communicating, Reflecting*) • identify events that occur every day eg 'We have news every day.' (*Reflecting*) • describe the position of the hands on an analog clock when reading hour time (*Communicating*)

(b)

Figure 1. Concept map and Early Stage One Time syllabus outcomes (NSWBOS, 2002).

Provided in Figure 2a is a concept map of Susan's interpretations of the Stage One syllabus outcomes (MS1.5 in Figure 2b). It illustrates that the main concepts are months, timing and telling time, which build upon and extend concepts already introduced in the previous stage.



(a)

Stage 1 — Time	
MS1.5 Compares the duration of events using informal methods and reads clocks on the half hour	**Key Ideas** Use informal units to measure and compare the duration of events Name and order the months and seasons of the year Identify the day and date on a calendar Tell time on the hour and half-hour on digital and analog clocks
Knowledge and Skills	**Working Mathematically**
Students learn about • estimating and measuring the duration of an event using a repeated informal unit eg the number of times you can clap your hands while the teacher writes your name • comparing and ordering the duration of events measured using a repeated informal unit • naming and ordering the months of the year • recalling the number of days that there are in each month • ordering the seasons and naming the months for each season • identifying a day and date using a conventional calendar • using the terms 'hour', 'minute' and 'second' • using the terms 'o'clock' and 'half-past' • reading and recording hour and half-hour time on digital and analog clocks	**Students learn to** • discuss activities that take one hour, less than an hour, more than an hour (*Communicating, Reflecting*) • indicate when it is thought that an activity has gone for one hour, one minute or one second (*Applying Strategies*) • solve simple everyday problems using problem-solving strategies, including: – trial and error – drawing a diagram (*Applying Strategies, Communicating*) • describe the position of the hands on a clock for the half-hour (*Communicating*) • associate everyday events with particular hour and half-hour times eg 'We start school at 9 o'clock.' (*Reflecting*)

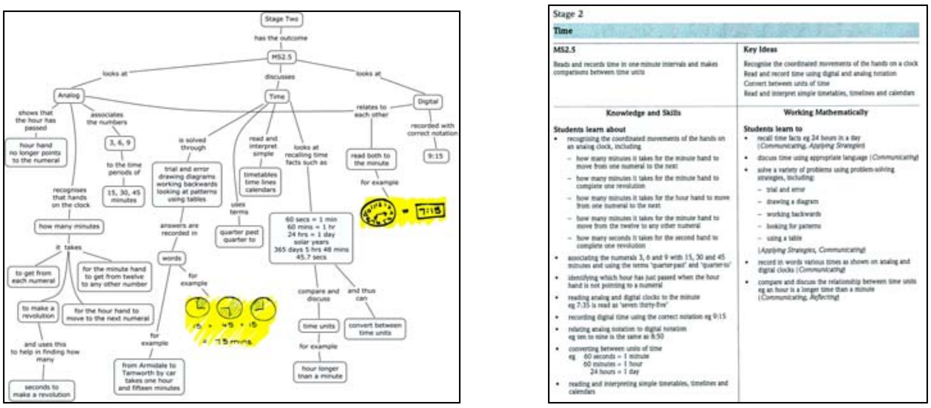
(b)

Figure 2. Concept map and Stage One Time syllabus outcomes (NSWBOS, 2002).

The leftmost branch describes the number of days per month, names and order of months with a right sub-branch describing that students record days and write dates using a conventional calendar. Another sub-branch (to the right) records the number of seasons, names and order as shown by the branch's illustrative example. For the timing branch in the middle, the propositions describe that timing means duration of time and it is estimated and measured using repeated informal units when two or more times are compared and ordered. An example of informal units is clap hands. The adjacent sub-branch (to the right) describes that students consider activities that take more or less than an hour so that they can indicate when an activity is completed in terms of hour, second and minute based on the knowledge that 60 minutes is one hour. Two (timing and telling time) of the main concepts are cross-linked through the linking words (problems involving are solved through) and connecting to two illustrative methods of solving problems, namely, trial and error and drawings and diagrams. The rightmost branch describes that dial reading means telling time; it uses and reads o'clock and half past, and describes that the half hour is the hour hand halfway between two hour markers such as shown on the analog clock for four thirty. More progressively differentiating links from the o'clock and half past node, on the far left, describes the meaning of on the hour in terms of the position of the minute and hour hands as illustrated by the analog clock showing four o'clock. The adjacent sub-branch to the right describes that telling time using o'clock and half past on analog and digital clocks and recording these involve the use of a variety of strategies as illustrated. The adjacent sub-branch to the right illustrates the use of o'clock and half past to identify everyday events such as on the bus at half past eight and at school at 9 o'clock. Overall, Stage One continues to consolidate the use of informal units to measure and compare the duration of events; introduces months, identification of day and date on calendar, and half hour; and extends understanding of the seasons of the year and telling time on digital and analog clocks to using both hour and half hour. A comparison of the concept map

(Figure 2a) to the text of the syllabus outcomes (Figure 2b) reveals that all key ideas and working mathematically strategies have been accommodated.

Susan's interpretations of the Stage Two syllabus outcomes (Figure 3b) showed that the main concepts are analog, time and digital as provided in Figure 3a. The leftmost sub-branch (Figure 3a) describes that, analog shows the hour has passed if the hour hand no longer points to the numeral. Four progressively differentiating links in the next sub-branch (to the right) illustrates that on the analog, the hands on the clock indicates how many minutes it takes to get from each numeral, to make a revolution and using the information to find how many seconds to make one revolution. The next link to the right focuses on number of minutes for the hour hand to move to the next numeral and the last link focuses on how many minutes for the minute hand to get from twelve to any other number.



Stage 2

Time

MS2.5	Key Ideas
Reads and records time in one-minute intervals and makes comparisons between time units	Recognise the coordinated movements of the hands on a clock Read and record time using digital and analog notation Convert between units of time Read and interpret simple timetables, timelines and calendars

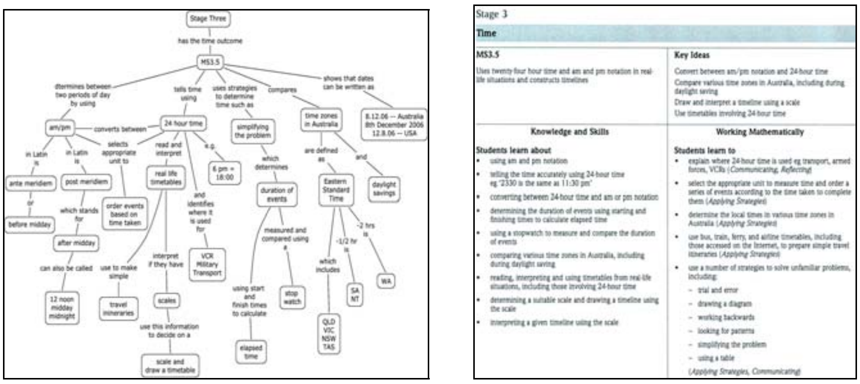
Knowledge and Skills	Working Mathematically
Students learn about • recognising the coordinated movements of the hands on an analog clock, including – how many minutes it takes for the minute hand to move from one numeral to the next – how many minutes it takes for the minute hand to complete one revolution – how many minutes it takes for the hour hand to move from one numeral to the next – how many minutes it takes for the minute hand to move from the twelve to any other numeral – how many seconds it takes for the second hand to complete one revolution • associating the numerals 3, 6 and 9 with 15, 30 and 45 minutes and using the terms 'quarter-past' and 'quarter-to' • identifying which hour has just passed when the hour hand is not pointing to a numeral • reading analog and digital clocks to the minute eg 7:35 is read as 'seven thirty-five' • recording digital time using the correct notation eg 9:15 • relating analog notation to digital notation eg ten to nine is the same as 8:50 • converting between units of time eg 60 seconds = 1 minute, 60 minutes = 1 hour, 24 hours = 1 day • reading and interpreting simple timetables, timelines and calendars	**Students learn to** • recall time facts eg 24 hours in a day (Communicating, Applying Strategies) • discuss time using appropriate language (Communicating) • solve a variety of problems using problem-solving strategies, including: – trial and error – drawing a diagram – working backwards – looking for patterns – using a table (Applying Strategies, Communicating) • record in words various times as shown on analog and digital clocks (Communicating) • compare and discuss the relationship between time units eg an hour is a longer time than a minute (Communicating, Reflecting)

(a) (b)

Figure 3. Concept map and Stage Two Time syllabus outcomes (NSWBOS, 2002).

At the top of the analog branch is a sub-branch describing the association of the numbers 3, 6, and 9 with the periods of 15, 30 and 45 minutes. The middle, time branch shows a left sub-branch describing that solving for time involves multiple methods such as illustrated where the answers are recorded in words (e.g., from Armidale to Tamworth by car takes one hour and fifteen minutes) or using analog clock diagrams and numbers as illustrated by the example. The sub-branch (to the right) describes the use of terms such as quarter past and quarter to with the adjacent sub-branch focusing on reading and interpreting simple timetables, time lines and calendars. The rightmost sub-branch of the time branch focuses on the recall of time facts such as provided in the illustrative example node, which enables conversion between time units. The sub-branch then extends further to compare and discuss time units, for example, (an) hour is longer than a minute. Further, a cross-link between the analog and digital nodes illustrates their interconnection when reading time to the minute such as shown by an analog clock and digital time of 7:15. The only other sub-branch from the digital node provides an example of the correct notation as 9:15. Overall, Stage Two introduces the concept of coordinated use of the hands of the clock to tell time, read and record time using digital and analog notations, convert between units of time, and read and interpret simple itineraries and calendars including an expanding repertoire of problem solving strategies.

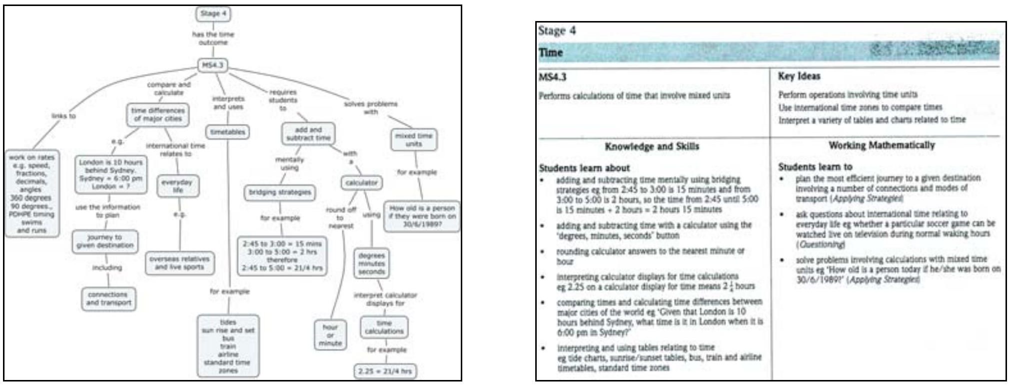
Susan's interpretations of Stage 3 syllabus outcomes are provided in Figure 4a. The main concepts are am/pm, 24 hour time, simplifying the problem, and time zones in Australia. The leftmost branch describes the distinction between two periods of the day as am/pm, which reads in Latin as ante meridiem or before midday and post meridiem or after midday, or alternatively labeled as 12 noon, midday or midnight. The meaning is not very clear with this last phrase of the proposition. According to the syllabus notes (not shown), midday and midnight need not be expressed in am or pm form; that is, 12 noon or 12 midday and 12 midnight should be used, even though 12:00 pm and 12:00 am are sometimes seen. A cross-link between the am/pm and 24 hour time nodes indicates that they can be converted to either form. Further, an integratively reconciliated connection to linking words (selects appropriate unit to), describes that events can be ordered based on time taken. The adjacent sub-branch (to the right under the 24 hour time node) describes its use to read and interpret real life timetables such as used to make simple travel itineraries; and to interpret, if they have scales, to decide a scale and drawing a timetable. The next sub-branch identifies where the 24 hour time is used. The final sub-branch provides an illustrative example, namely, 6 pm = 18:00. The concept map further illustrates that the MS3.5 outcomes (Figure 4b) include different strategies to determine time, such as simplifying the problem to determine the duration of events by using start and finish times to calculate elapsed time and that times can be measured and compared using a stopwatch. The next branch (to the right) describes time zones in Australia and daylight savings. Also time zones are defined as Eastern Standard Time (EST), which includes the listed four states Queensland (QLD), Victoria (VIC), New South Wales (NSW) and Tasmania (TAS). The adjacent sub-branch (to the right) indicates South Australia (SA) and Northern Territory (NT) are behind EST by a half-hour while Western Australia (WA) is behind two hours. Overall Stage Three outcomes focus on the use of the 24 hour time and am/pm notation and conversion between them and construction and interpretation of time lines using a scale.



(a) (b)

Figure 4. Concept map of the Stage Three Time syllabus outcomes.

The Stage Four concept map (Figure 5a) illustrates the link to work on rates such as speed, fractions, decimals, angles, degrees and PDHPE timing of swims and runs. PDHPE, Personal Development, Health and Physical Education, is one of the Key Learning Areas in New South Wales, Australian schools.



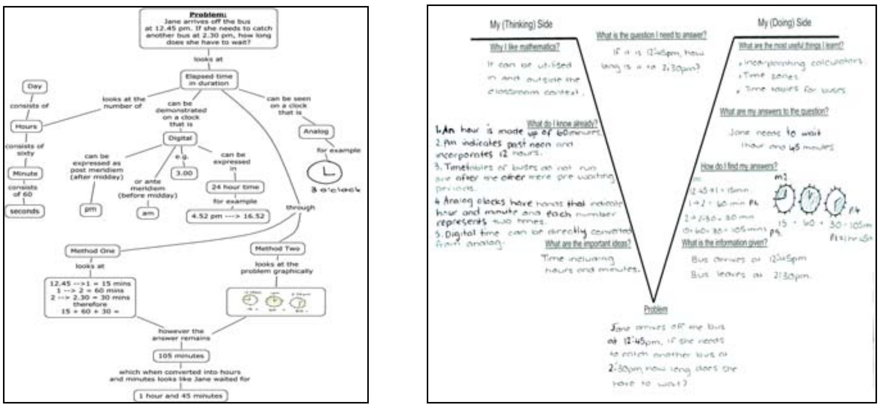
(a) (b)

Figure 5. Concept map of the Stage Three Time syllabus outcomes.

Also the focus is on comparing and calculating time differences of major cities such as the problem posed as an illustrative example. The proposition extends to indicate that such information can be used to plan journeys to given destination including connections and transport. Time differences of major cities, that is, international time relate to every day life; for example, (for contacting) overseas relatives and (determining times for) live sports. The map further illustrates that MS4.3 involves the interpretation and use of timetables such as tides, sun rise and set, bus, train, airline, and standard time zones. The next branch involves the addition and subtraction of time using mental bridging strategies such as illustrated for the time elapsed between 2:45 to 5:00, and, calculator where answers may be rounded off to the nearest hour or minute. The right sub-branch of the calculator node describes that degrees, minutes and seconds may be used to interpret calculator displays for time calculations, for example, 2.25 = 21/4 hrs. Overall, Stage Four focuses on the performance of operations involving time units, use of international time and time zones to compare times, and the interpretation of a variety of tables and charts related to time.

4 Mathematics Problem Vee Diagram and Concept Map

Provided in this section are Susan’s conceptual analyses of a mathematics problem using both a concept map and a vee diagram. The analysis is in terms of the prior knowledge and mains concepts required to solve the problem and the methodological aspects such as the given information, methods of solving the problem and answer to the focus questions. The results are displayed on a vee diagram (see Figure 6b). Also provided is a concept map of the problem (Figure 6a), illustrating the main concepts Susan expected students to understand and apply in determining the elapsed time as well as an overview of two methods of solving the problem. The given problem statement is positioned at the tip of the vee diagram (under *Problem*) with a focus question (If it is 12:45 pm, how long is it to 2:30 pm?) which Susan crafted based on her interpretation of the problem and what is needed to be obtained. On the left side of the vee (under *What do I know already?*) is a list of principles Susan anticipated a student would require in determining possible solutions. On the right side (under *How do I find my answers?*) are two methods of solving the problem. As well as providing the main steps, Susan also cross-referenced the appropriate principles (from the left side of the vee) as mathematical justifications for the steps on the right side. For example, principle 1 (P1: An hour is made up of 60 minutes) is used to justify the step: 1 → 2 = 60 mins (Line 2) of Method 1 (M1) while principle 5 (P5: Digital time can be directly converted from analog) is used to justify the steps: 2 → 2:30 = 30 min (Line 3), and 15 + 60 + 30 = 105 mins (Line 4, M1). In comparison, for Method 2 (M2), principle 4 (P4: Analog clocks have hands that indicate hour and minute and each number represents two times) is used to justify the times shown on the 3 analog clocks (Line 1), and 15, 60 and 30 minutes = 105 mins (Line 2) whilst principle 1 (P1) is used to justify the conversion from 105 mins to 1 hr 45 mins (Line 3). These principles appear directly on the opposite side (to the methods) thus reinforcing the close correspondence between the principles (propositions) and their application as justifications for the steps in the two methods.



(a) (b)

Figure 6. Concept map and vee diagram of a mathematics problem.

Also provided in Figure 6b, is an answer to the focus question posed, namely, Janes needs to wait 1 hour and 45 minutes (under *What are my answers to the question?*). Listed under *What are the most useful things I learnt?* at the top right side of the vee, are Susan's planned sub-topics for subsequent learning. Specifically, the use of calculators as flagged in Figure 5a (at Stage Four) while time zones were first introduced in Stage Three (Figure 4a). With the third sub-topic, simple timetables were initially introduced in Stage Two (Figure 3a), extended upon in Stage 3 (Figure 4a) before progressing to more complex timetables in Stage Four (Figure 5a). This simple to complex progressive trend is visible by tracing the occurrence of the timetable concept in the concept maps from Figures 3a through Figure 4a to Figure 5a. This section of the vee reflects Susan's future pedagogical intentions to extend the knowledge and understanding students gained by solving the problem. On the left side of the vee (under *Why I like mathematics?*), are Susan's expectations of what students would inevitably find for themselves as a result of engaging with such problems. Whilst only one vee diagram is presented here to illustrate its application in problem solving, many more were constructed to highlight the recursive application of different combinations of propositions from the five concept maps on time as justifications of multiple solutions to a problem. However, space constraints do not allow them all to be presented.

5 Journal of Reflections

The impact of constructing maps/diagrams on Susan's mathematical thinking, reasoning and pedagogical planning was documented in her journal of reflections during the semester (excerpts are in italics). Her main reason for taking the unit was to help her develop a better understanding of the mathematics syllabus and of the teaching of its concepts to primary students. Rather than the year twelve (end of secondary level) style that she "*was used to, of formulas are everything*", she perceived the need to approach primary mathematics from a primary perspective. In so doing, she felt she "*was taking a major risk in that (she) was approaching something that would take apart all (her) previously attained ideas and approaches*." Prior to the study, Susan viewed problems as simply questions to be answered and topics as containing a lot of information around one idea that needed to be taught to students. However, upon completing concept mapping and vee diagramming activities, it became increasingly clear that "*there was more to a problem than a formula and an answer*." Instead, "*problems consisted of a wide variety of factors that contributed to the understanding and subsequent answer*" such as the kinds of prior knowledge one possessed, which influenced the methods, and through reflection, the value of the learning experience, subsequent learning or extensions to the current activity. Consequential to constructing and completing more vee diagrams (similar to the example vee diagram provided), Susan realised that solving a problem became more than just "*an answer finder*". Initially, she found it difficult to complete the thinking side because "*(she) did not know how (she) constructed the answer on the right hand side …and thus, did not know what principles (she) had to list nor the important ideas*". However, she wrote, *"I struggled with this as, as a student I had only been taught the formulas never what was behind them.*" With this self-realisation, Susan chose to challenge herself, namely, "*before finding the answer in future diagrams, first, (she) would look only at the question and think what (she needed) to know about it before (she actually solved) the problem.*" Susan further admitted it was always difficult for her to explain problems to others, "*I always had difficulty in explaining what I wanted them to do and it frustrated me that they did not understand when I explained it the first time.*" Through her reflections though, she said that her "*communication skills verbally (had) been assisted greatly by (her) written communication in both concept maps and vee diagrams.*" She claimed, "*I now have the basic skills written before me and because it was me that had to construct the written version I was able to explain what I did verbally better than I had done before.*" Consequently, through concept mapping syllabus outcomes, Susan eventually realised that a topic has a number of key and relevant concepts and recommended strategies that should be incrementally introduced, consolidated and extended for students through a suitable selection of learning activities to ensure the development of a conceptual understanding of the topic.

6 Discussion & Implications

Findings suggested Susan became competent and confident in her critical abilities to analyse syllabus outcomes and a problem and to display the results appropriately on concept maps and a vee diagram respectively. Critically analysing the list of syllabus outcomes at each stage for key concepts, working mathematically strategies and illustrative examples, she positioned the results in a hierarchy of interconnecting network of propositions. The resulting 5 concept maps collectively explicated visually and through propositions, a developmental trend for the teaching of key concepts and strategies. Initially introduced as single concepts, the main ideas became increasingly more complex and interconnecting with other ideas progressively through the stages. For example, the idea of telling time was introduced by reading the hour on digital and analog clocks in early stage one, consolidated again in stage one and extended to include the half hour. The descriptive language used also expanded from o'clock to include half-past. By stage two, telling time expanded to reading time to the minute to include 15 minute intervals with the introduction of the 24 hour and am/pm notation in stage three with the descriptive language expanding to quarter to and quarter past. Increasingly it became clear that mapping the syllabus outcomes (compared to reading from a sequential list as provided in the syllabus) has its advantages. For example, it facilitated a big picture view of key ideas and recommended strategies and reinforced the recursive but increasingly expanded nature of key ideas with each stage, which would enable a teacher to quickly identify potential pedagogical points of insertion for further developmental work be it introductory, consolidation, remediation or extension for meaningful learning. Through her maps/diagram, Susan communicated effectively with her audience. Because she had individually constructed them and actively engaged with the critical processes of analyzing and making connections, she was in a better and stronger position to explain and justify her ideas publicly. Susan further realised she was able to "*see the connections that infiltrated the topic*", consequently gaining a better understanding of sequencing learning activities. For example, "*(she) now understands what needs to be taught first and where (she) needs to go from there*" through the connections she made visible on maps and planned for on a vee diagram. Further, over time, completing the thinking side of a vee diagram eventually became much easier and done as efficiently as she did the doing side. At times, she challenged herself using the thinking side first to guide the development of methods, which was something she did not use to do before. This was a significant development in her critical approach to problem solving. For example, she recorded that the principles guided her development of appropriate solutions, and sometimes, if the method is done first, she could flexibly use the solutions to infer what the principles should be.

The presented data focused on time syllabus outcomes and formed part of Susan's work on the measurement strand over the semester. The maps/diagram explicitly illustrated the richness of information that can be captured by the combined usage of maps and a diagram in analysing syllabus outcomes for the former and making explicit relevant propositions guiding multiple solutions of a problem with the latter. Susan's maps/diagram demonstrated that a deep understanding of the time sub-strand was developed and reinforced through their construction. The vee diagram structure provided not only the space to express one's mathematical beliefs and critical reflections, but also projections for future learning. Overall, constructing maps/diagram evidently encouraged Susan to move beyond a procedural view to a more conceptually based justification of methods and a purposeful and clearer understanding of sequencing prior, new and future learning to promote students' developmental and conceptual understanding. Susan also became increasingly confident in her abilities and skills to use her concept maps critically as a means of identifying subsequent learning and the next developmentally appropriate method of solution guided by the networks of propositions. Finally, Susan concluded that constructing maps/diagram had begun "*a new chapter in (her) understanding and teaching of mathematics.*" She felt confident and her understanding of the sub-strand had deepened particularly in "*how each and every one of (the concepts and strategies) builds upon the prior knowledge of the last*". Findings from this case study contribute empirical data to support the use of maps/diagram to develop primary teachers' deep understanding of syllabus outcomes and the pedagogical use of concept maps to make explicit the developmental trends of key ideas across multiple stages and a vee diagram to highlight the critical synthesis of conceptual and methodological knowledge in problem solving and the importance of explaining, justifying and validating multiple solutions mathematically in terms of stage-appropriate propositions. The visual displays of networks of propositions on concept maps and theoretical and procedural information of a problem on a vee diagram effectively encapsulated the interconnection between the Knowledge & Skills and Working Mathematically Syllabus Outcomes. As the data demonstrates, constructing concept maps engenders a deep understanding of how concepts and recommended working mathematically strategies are developmentally progressed, consolidated and extended across the stages while constructing a vee diagram enhances the critical synthesis of the relevant mathematical principles and methods of generating solutions to a problem. Collectively the impact engenders a better pedagogical appreciation of how the main concepts and recommended strategies are developmentally progressed within each stage, and how its mathematical principles are applied in methods. These findings imply that concept mapping is a potentially useful strategy for unpacking the mathematics underpinning syllabus outcomes and making explicit developmental trends to facilitate pedagogical planning for teaching and designing learning activities while completing a vee diagram is a viable strategy for challenging students' critical thinking, reasoning and synthesis as they determine multiple solutions to a problem. The applications of these tools in the classroom as teaching tools for the teacher and consequently, the impact of this preparatory work on student learning are areas worthy of further investigation.

References

Afamasaga-Fuata'i, K., (2007). Using concept maps and vee diagrams to interpret '*area*' syllabus outcomes and problems. In *Mathematics: Essential Practice.* In K. Milton, H. Reeves, & T. Spencer (eds), Mathematics essential for learning, essential for life. Proceedings of the 21st biennial conference of the Australian Association of Mathematics Teachers, Inc., pp. 102-111. University of Tasmania, Australia, AAMT.

Afamasaga-Fuata'i, K., & Reading, C. (2007). Using concept maps to assess preservice teachers' understanding of connections between statistical concepts. Published on IASE site, http://www.swinburne.edu.au/lss/statistics/IASE/CD_Assessment/papers/IASE_SAT_07_Afamasaga_Reading.pdf

Afamasaga-Fuata'i, K. (2006). Innovatively developing a teaching sequence using concept maps. In A. J. Cañas & J. D. Novak (Eds.), *Concept Maps: Theory, Methodology, Technology. Proceedings of the Second International Conference on Concept Mapping,* Volume 1, (pp. 272-279). San Jose, Costa Rica: Universidad de Costa Rica.

Afamasaga-Fuata'i, K., (2005). Students' conceptual understanding and critical thinking? A case for concept maps and vee diagrams in mathematics problem solving. In M. Coupland, J, Anderson & T. Spencer (eds). *Making Mathematics Vital.* Proceedings of the Twentieth Biennial Conference of the Australian Association of Mathematics Teachers (AAMT), (pp. 43-52). January 17 – 21, 2005. University of Technology, Sydney, Australia.

Afamasaga-Fuata'i, K. (2004). Concept maps and vee diagrams as tools for learning new mathematics topics. In A. J. Cañas, J. D. Novak & F. M. González (eds). *Concept Maps: Theory, Methodology, Technology*. Proc. of the First Int. Conference on Concept Mapping September 14-17, 2004 (pp. 13 – 20). Pamplona, Spain: Univ. Pública de Navarra.

Afamasaga-Fuata'i, K. (1998). *Learning to Solve Mathematics Problems Through Concept Mapping & Vee Mapping.* National University of Samoa.

Ausubel, D. P. (2000). *The Acquisition and Retention of Knowledge: A Cognitive View*. Dordrecht; Boston: Kluwer Academic Publishers.

Brahier, D. J. (2005). *Teaching secondary and middle school mathematics.* Second Ed. New York: Pearson Education, Inc.

Jonassen, D.H., Peck, K.L., & Wilson, B.G. (1999) *Learning with technology*. Upper Saddle River, NJ: Merrill Publishing.

Hannson, O. (2005). Preservice teachers' views on $y = x + 5$ and $y = \pi x^2$ expressed through the utilization of concept maps: A study of the concept of function. Retrieved on March 3, 2008 from http://www.emis.de/proceedings/PME29/PME29RRPapers/PME29Vol3Hansson.pdf.

Mintzes, J. J., Wandersee, J. H., & Novak, J. D. (Eds.) (2000). *Assessing science understanding: a human constructivist view.* San Diego, California, London: Academic.

New South Wales Board of Studies (NSWBOS) (2002). *K-6 Mathematics Syllabus*. Sydney, Australia: NSWBOS.

Novak, J. D. & Gowin, D. B. (1984). *Learning How to Learn*. Cambridge University Press.

Williams, C. G. (1998). Using concept maps to assess conceptual knowledge of function. *Journal for Research in Mathematics Education, 29,* (4), 414-421.

Wiske, M.S. (1998). What is teaching for understanding? In M.S. Wiske, (Ed.) *Teaching for understanding: Linking research with practice*. San Francisco: Jossey-Bass Publishing.