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MAPPING CANTOR'S DUST: MATHEMATICAL UNDERSTANDING OF NURSING STUDENTS

James J. Vagliardo, Jean Schmittau
State University of New York at Binghamton, USA

Abstract. Nursing education literature documents the extent to which student nurses fail to correctly use mathematics to accurately complete such tasks as unit conversions, dosage calculations, and fluid monitoring. Guided by "The Maths for Nurses" taxonomy established by Pirie (1987), a five part protocol was developed and used as the basis for semi-structured clinical interviews with 24 student nurses. Concept maps were used in analyzing the results. These revealed the nature of the conceptual knowledge of the student nurses which, for a significant number, was sufficiently fragmented to merit the designation "Cantor's Dust" (Gleick, 1987). In the absence of meaningful relational connections and integrated cross linkages, the student nurses failed to correctly apply mathematics to nursing in a wide variety of clinical contexts.

1 Introduction

Student nursing programs have been reporting significant mathematical deficiencies among their populations for nearly four decades. Despite various intervention approaches the problem lingers and otherwise excellent candidates for the nursing profession are eliminated for this lack of mathematical competency. The mathematics employed by nurses using the international metric/SI system of weights and measures in medicine is not complicated. Nursing tasks such as unit conversions, dosage calculations, and fluid monitoring each depend on the set up and simplification of expressions that involve notions of ratio and proportion and a good understanding of rational numbers expressed in both fractional and decimal forms. Substantive understanding of these concepts render such calculations routine. Yet, in a 1979 Michigan case study of nursing errors in drug administration, Brown reported that wrong dose and rate of infusion mistakes accounted for 33% of the reported errors and were likely the result of mathematical miscalculation (cited in Pirie, 1987, p.14). The severity of miscalculation is evident in the simulation study published in the *American Journal of the Diseases of Children* the same year. Perlstein et al., whose study involved the staff of a neonatal intensive care unit working with simulated physician's orders, reported that "56% of the errors tabulated would have resulted in administered doses ten times greater or less than the ordered dose" (cited in Pirie, 1987, p.145.) Recent studies indicate that the problem persists. Gillham and Chu (1995) found that only 55% of 158 second year nursing students in their study correctly answered ten questions of common clinical calculations. Lesar (2003) studied and classified 200 tenfold errors in medication dosing that occurred in an 18 month period at a 631-bed teaching hospital, citing such errors as a misplaced decimal point, adding an extra zero, or omitting a necessary zero.

2 Evaluating Student Nurses' Meaningful Understanding of Mathematics

2.1 Conceptual Perspective

Pirie (1987) established a comprehensive taxonomy of mathematical skills required in nursing. Considerable research of student nurses' mathematical deficiencies has been focused on their ability to correctly perform these standard arithmetic operations and apply them to nursing. However, nursing education research has yet to adequately consider the problem from a *conceptual* perspective. The study reported here focused on the nature of the conceptual knowledge student nurses possess regarding the mathematics necessary for nursing. A representative sample of twenty-four student nurses engaged in clinical interviews on a five-part protocol ranging from basic mathematical understandings foundational for clinical practice to actual clinical applications. Those who had learned basic mathematical procedures as rote processes did not have the integrated, relational knowledge necessary for correct and reliable application of mathematics to nursing science.

2.2 *Theoretical Framework*

If student nurses understand mathematics as a conceptual system, their thinking will indicate a hierarchical structure of ideas with specific ideas subsumed by more inclusive mathematical thought. There will be evidence of ample relational linkages between concepts sufficient to allow for flexible reasoning in problem solving situations. Integrated thinking will be possible across the full scope of mathematical ideas salient to nursing practice.

Ausubel's (1968) discussion of the difference in cognitive effect of meaningful learning in comparison to rote learning is important to consider in attempts to clinically identify the quality of conceptual understanding a study participant may reveal. As a consequence of meaningful learning, a student will have the "availability of relevant and proximately inclusive subsumers that provide clarity, discriminability, cohesiveness, and integration of thought" (p. 136). Ausubel posits that, as meaningful understanding develops, each new concept gets "anchored" into the existing cognitive structure and the processes of *progressive differentiated* and *integrative reconciliation* "make possible the acquisition of abstract ideas in the absence of concrete empirical experience " (p. 507). This is not the case in rote learning where recall is of primary importance and constitutes the only learning goal. In order to further the memorizability of new information, the consequence of rote learning is the establishment of "arbitrary associative bonds between discrete, verbatim elements, isolated in an organizational sense from established ideational systems …." (p 109). In Ausubel's view, rote learning actually "resists progressive assimilation" while meaningful learning seeks "appropriate relational anchorage within a relevant ideational system" that involves those features that concept maps are designed to reveal. Skemp (1976) has described this difference in his work in mathematics education, referring to the result of rote learning as "rules without reasons" and the result of meaningful learning as "relational understanding." Therefore, a comprehensive, well developed conceptual schema is one in which mathematical knowledge is relationally linked in multiple ways

2.3 *Betweenness as an Indicator of Meaningful Conceptual Connection*

Ausubel (1968) discusses the difficulty of demonstrating that meaningful learning has occurred and makes suggestions on how best to proceed in a clinical setting:

> If one attempts to test for such knowledge by asking students to state the critical attributes of a concept or essential elements of a proposition, one may merely tap rotely memorized verbalizations. At the very least, therefore, tests of comprehension must be phrased in a different language and must be presented in a somewhat different context than the originally encountered learning material. (Ausubel, 1968, p.110).

Ausubel was concerned that direct efforts to evaluate meaningful learning might be compromised by students' extensive test taking experience and their adeptness at memorizing key ideas and formulas. To avoid the substitution of rote memory for meaningful comprehension, Ausubel advises "posing problems that are both novel and unfamiliar in form and require maximal transformation of existing knowledge" (1968, p.111). This advice was applied to the creation of nine problems which required the identification of a decimal whose value lies between two given numbers. The pairs of numbers given were: 1/4 and 3/4, 3/4 and 7/8, 1/2 and .51, 1.715 and 1.716, 3/10 and 1/3, 1/9 and 2/9, 75% and 75/60, one billion and one billion one, and two millionths and five millionths. Safety in estimating, accuracy in calculating, and error free administration of medications depend on the mindful application of the processes of converting fractions to decimals, decimals to fractions, and the meaning of decimal place. Consequently, the analysis of "think-aloud" responses provided the first basis of evaluation of students' conceptual understanding of the mathematics of nursing.

The "betweenness" problems were designed to evoke the mathematical thinking of student nurses at a conceptual level, revealing meaningful connections. The unusual question format differed from typical mathematical problems presented to students. The design intentionally makes problems more difficult for students with isolated procedural knowledge and favors those with connected relational knowledge. In finding decimal values between the pairs of numbers, a student who recognized the relationships between several mathematical ideas was more successful. These students reasoned their way to a correct answer based on the interconnectedness of their understanding. The rational numbers presented in various forms (i.e. fractions, decimals, percents) encouraged students to reveal meaningful associations. The intent was to avoid questioning that directed a respondent to perform a particular arithmetic skill. By incorporating the need for fraction, decimal, and percent conversions within the task of locating a rational number between two others, participants were required both to select their own method (rather than be directed by the question itself), and to reason mathematically across the various rational representations.

Given adequate conceptual understanding of mathematics, correct answers to the nine problems in section 1 can be found quickly and easily. Student thinking that revealed a connected conceptual system of mathematical thought, generated correct answers through quick, mental reasoning. Conceptual discontinuity was associated with complicated methods that involved a struggle to find a valid answer. When a student's conceptual understanding consisted of segmented knowledge, correctness was often compromised and calculator dependency was evident. Results showed that only 38% of the answers provided by nursing students had the combined qualities of being mentally determined, and done quickly and correctly.

3 Conceptual Analysis through Concept Mapping

3.1 Conceptual Essence of Betweenness

Two mathematical ideas figure prominently in the betweenness thinking requested of the student nurses, the density of the real number line and the ordering relation. "Greater than" and "less than" comparisons are required in order to determine if a value in question is between two distinct real numbers (i.e. x is between a and b iff $x>a$ and $x<b$, where $a<b$). Comparision necessarily includes an understanding of the decimal system as positional. Knowledge of the nature of each decimal position provides the relational crosslink that enables fraction to decimal and decimal to fraction conversions. Adeptness with conversion algorithms alone are insufficient for the nine betweenness problems. The task requires conceptual connectedness

3.2 Deficiencies in Conceptual Connectedness

Study participants had difficulty considering what value might exist between 0.50 and 0.51, 1.715 and 1.716. Students remarked that there are no numbers between these pairs or determined only a finite number of decimal values. The fractional pairs 6/8 and 7/8, 1/9 and 2/9 gave evidence of the same confusion. The subsuming concept is fraction. The task is essentially locating the numbers 1/9, 2/9, 1/4, 3/10, 1/3, 1/2, 51/100, 3/4, 7/8, 75/60, 1715/1000, 1716/1000 on the number line (in this order) and then finding other values in various intervals of these locations. Decimals are particular instances of fractions (i.e. those fractions whose denominators are powers of ten). Converting to equivalent decimals 0.111... , 0.222... , 0.25, 0.3, 0.333... , 0.5, 0.51, 0.75, 0.875, 1.25, 1.715, 1.716 can provide a common basis for comparision. However, the relational understanding that makes knowledge of numbers meaningful for some students was insufficient to support acceptance of the density of the real numbers on the number line. The discontinuity in students' conceptual structure, was reflected in their view of the real numbers, which themselves were seen as discontinuous. The following student remarks are characteristic of the difficulties that arose when there were no meaningful connections between mathematical concepts.

Problem 2 : 3/4 and 7/8.
[Having changed 3/4 to 6/8 Lucy can not find a fraction between 6/8 and 7/8.]
Lucy: So I would have to do between six things and seven things. [silence] I'm trying to do it with fractions but I don't know how to. [laughs]

Problem 3 : 1/2 and 0.51.
[Rickie fails to find a decimal value between 1/2 and 0.51 after three tries]
Rickie: In between these is zero point five, zero point three one, zero point [pause] forty two. Forty two.

Problem 4 : 1.715 and 1.716.
[Nel does not understand the completeness of the real number line.]

Interviewer: How many numbers are between 1.715 and 1.716?
Nel: Just one.
Interviewer: How about between 1/4 and 3/4?
Nel : About ten.

Problem 5 : 3/10 and 1/3.
[Katelyn makes incorrect connections between fractions and the decimal system]
Student : 3/10 would be like [pauses], almost 1/3. So I would go between, well 1/3 is 0.3333 . So I would put 0.3334 [laughs] and just bring it out."

Problem 6 : 1/9 and 2/9.
[Mandy does not understand the completeness of the real number line.]
Interviewer: How many numbers are between 0.1111... and 0.2222... ?
Mandy: Do you want me to count them?
Interviewer: Answer any way you want to.
Mandy: There are 10. Is that right? 10 one hundredths [voice fades off to silence] on the other side of the decimal.

3.3 Problem 2: Write a decimal number whose value is between 3/4 and 7/8.

The analysis of problem 2 presented here is representative of the analysis carried out on each of the betweenness problems. Problem 2 generated considerable insight into the mathematical understandings of the student nurses. Answers were the result of 18 different solution paths and 11 of the 24 responses required the use of a calculator. Despite access to a calculator there were 5 incorrect responses recorded for an error rate of 21%. Analysis through concept mapping revealed the requisite understandings involved in problem 2 and the nature of students' meaningful understanding of the required concepts (Figure 1).

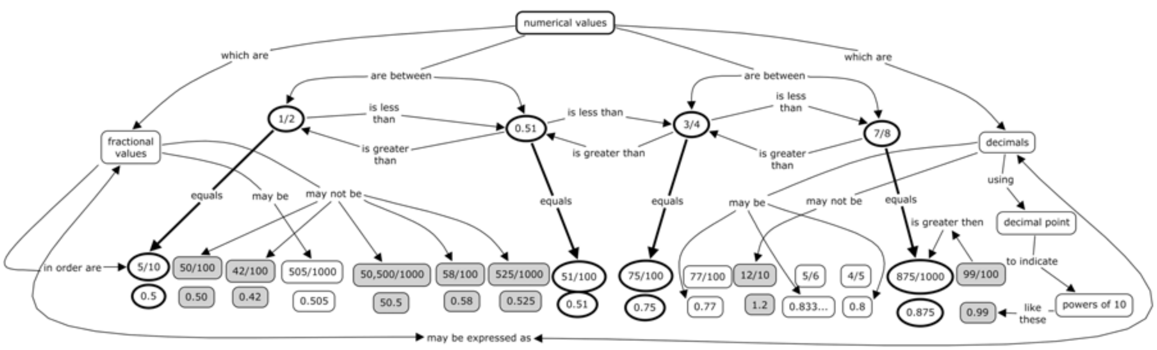


Figure 1. Concept map related to Problems 2 and 3: Write a decimal number whose value is between 3/4 and 7/8, and one between 1/2 and 0.51, respectively. Incorrect student values are shaded.

Deficiencies in conceptual connectedness were marked by an insufficient knowledge of decimal place, awkward numerical work-arounds, and invalid, unsupported mathematical reasoning. The conceptual deficiency was compounded by a limited command of number facts and an associated dependency on calculator support. Expressed insecurity about answers and methods was frequent. Alternative methods of checking for errors or resolving confusion were seldom recognized. Ignoring digits and decimal points further compromised meaningful conceptual connection. The decimal values displayed in order in figure 1 represent specific points on the number line that are relevant to the mathematical reasoning student nurses used in determining a decimal value between 3/4 and 7/8, and between 1/2 and 0.51. Students with meaningful knowledge of the number system required no calculation. A few simple number comparisons were sufficient. Students who did not understand numbers as a part of a relationally connected conceptual system struggled. A typical approach was to convert 3/4 to 6/8 and then try to locate a value "half way" between 6/8 and 7/8. Frequently hesitating at this point, expressions like 6.5/8 (i.e. 13/16) were identified and converted to their decimal equivalence by a division, done on paper or using the calculator. One student, Rickie, thought 5/6 was "half way" because 5 is midway between the original numerators 3 and 7 and 6 is midway between the denominators 4 and 8. 5/6 is between 3/4 and 7/8, but not the midpoint.

The work of Somer, Patricia, Rickie, and Myra further illustrate the mathematical reasoning of student nurses with respect to decimals and fractions. Somer's work on problem 2 was quick, confident, and done mentally. She knew 3/4 to be 0.75, determined 7/8 to one decimal place by a partial division, and chose 0.77 as an answer, confirming on the basis of her knowledge that "7/8 is greater than point eight." Patricia followed the same line of reasoning with somewhat less precision. She too knew the decimal equivalent of 3/4 but her estimate of 7/8 to be 0.99 left open the possibility of error since numbers in the interval [0.875..0.99) would not correctly answer the problem. Patricia correctly chose 0.8 as a value between 3/4 and 7/8, but her method was risky. "It just jumped out at me," was Rickie's explanation. It appeared that she chose a numerator between 3 and 7, and a denominator between 4 and 8, and constructed the fraction 5/6 which is in fact between 3/4 and 7/8. The use of the calculator (Rickie admits to being calculator dependent) provides post facto confirmation that 5/6 as a correct answer but only by chance. It can be shown mathematically that only proportional selections of numerators and denominators will produce numbers between 3/4 and 7/8; other selections do not. Like Patricia's poor estimation of the decimal equivalent of 7/8, Rickie's reasoning is risky, and disconnected from meaningful understanding of the number system.

Myra's response to this problem was incorrect. Myra's work in general was marked by precise knowledge of calculation procedures. Unlike Rickie's dependency, Myra avoided calculator use, preferring to work out problems using paper and pencil. Her careful, clearly legible work on this problem showed that there was no weakness in her ability to calculate, but her answer of 1.2 is unreasonable since neither 3/4 nor 7/8 is greater than 1. Myra converted 3/4 to 6/8 and correctly determined that (6.5)/8 is a value between 6/8 and 7/8. In expressing her answer as a decimal, she reversed the order of the division, dividing 6.5 into 8 rather than 8 into 6.5. Myra's well conditioned arithmetic skill is an example of the effectiveness of rote learning to provide "instrumental understanding" (Skemp, 1976) while missing the relational connection that would have avoided error, the fact that the answer must be less than 1. Analysis of student responses to problem 3 reflected inadequacies similar to those encountered in problem 2 (see figure 1).

3.4 Additional Commentary on Betweenness Problems

Students' lack of knowledge of the decimal equivalence of the ninths (1/9 = 0.1111… , 2/9= 0.2222… , see figure 2) eliminated locating 7/9 as a value between 3/4 and 7/8. Furthermore, some students do not differentiate between the finite decimal 0.77 and the repeating decimal 0.7777… , which would mean that 77/100 = 7/9, which is not true. Recalling Ausubel's notion of progressive differentiation as a quality of conceptual understanding, it is clear that student nurses who do not see the distinction between different rational numbers will have difficulty applying mathematical reasoning that depends on relational understanding. Estimating 7/8 as 0.99 reflects the potential for error in locating a numerical value in a required interval of a number line. This is the essence of what student nurses must do when they are using scaled nursing equipment or rescaling a prescription dosage to a different unit of measure. Numerical knowledge must be integrated in such a way as to make meaningful sense of measurement. For students who understand the real numbers as a conceptual system, knowledge that 5/7 is not between 3/4 and 7/8, and that 7/9 is, should be easy to determine.

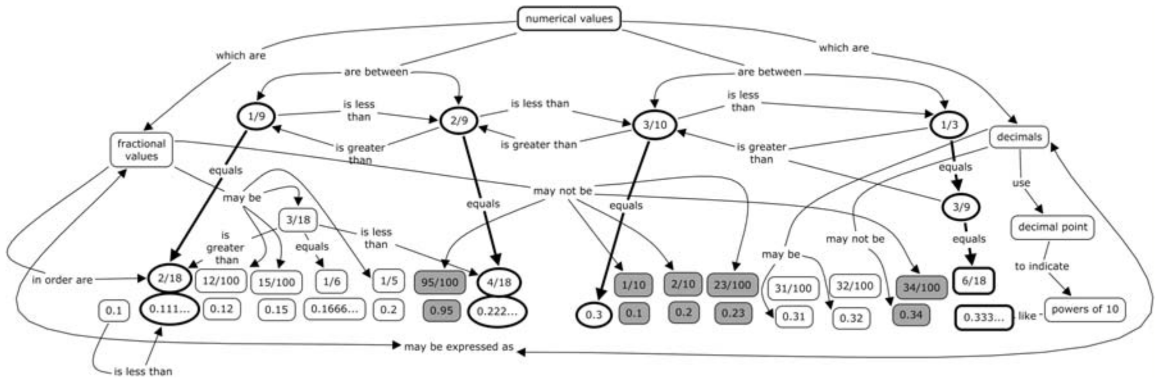


Figure 2. Concept map related to Problems 5 and 6: Write a decimal number whose value is between 3/10 and 1/3, and one between 1/9 and 2/9, respectively. Incorrect student values are shaded.

Only 9 students correctly identified a decimal value between two millionths and five millionths. The other 15 students had no meaningful understanding of decimal positions beyond thousandths. Student values ranged through three orders of magnitude beyond the endpoints of the interval (between 0.3 and 300 millionths for an interval with endpoints 2 millionths and 5 millionths. See figure 3).

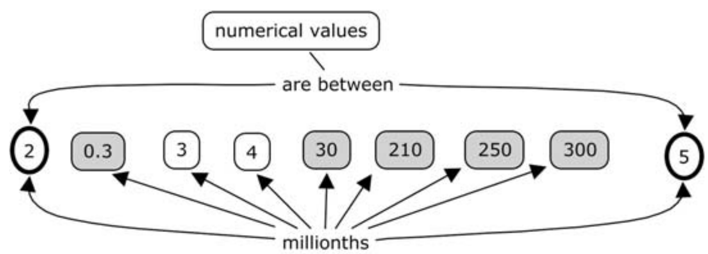


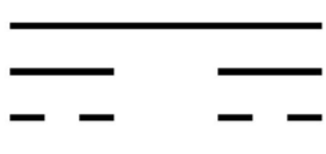
Figure 3. Concept map rescaled in millionths, related to Problem 9: Write a decimal number whose value is between two millionths and five millionths. Incorrect student values are shaded.

4 Findings

4.1 Cantor's Dust

Cantor's Dust refers to the mathematical construction that emerged from the set theory work of nineteenth-century mathematician Georg Cantor. The Cantor Set is constructed by removing the middle third of a line segment, followed by repeated iterations on each resulting segment, ad infinitum. Smaller and smaller line segments remain after each removal with the result that while infinitely many line segments remain, what began as a segment becomes everywhere discontinuous. In short, the clusters of remaining points appear like dust. (See below).

> To make a Cantor Set you start with the interval of numbers from zero to one, represented by a line segment. Then you remove the middle third. That leaves two line segments, and you remove the middle third of each (from one-ninth to two-ninths and from seven-ninths to eight-ninths). That leaves four segments, and you remove the middle third of each, and so on to infinity. What remains? A strange "dust" of points, arranged in clusters, infinitely many yet infinitely sparse.
> (Gleick, 1987, p. 92)



4.2 *Findings of Conceptual Disconnection*

The nature of a significant number of the student nurses' conceptual understanding of mathematics resembled Cantor's Dust, fractalized and discontinuous. In its most severe form of discontinuity, meaningful conceptual relationships between fractions, decimals, percents, and proportional procedures, either did not exist, were incorrect, or were inadequately developed. In the absence of meaningful conceptual connection, student nurses relied on isolated knowledge of arithmetic methods, formulaic approaches, and the assumed infallibility of calculating devices. Accurate and reliable numerical calculation, precise use of medical equipment, correct interpretation of graphical information, and success at applied problem solving proved difficult. In short, these students were unable to work with the metrics of nursing confidently and without serious error. The lack of density of known values along the number line, the disjoint nature of conceptual connection, and lack of relational integration is indicative of the findings of conceptual disconnection among student nurses.

4.3 *Nursing Consequences of Conceptual Disconnection*

Students' lack of meaningful understanding of the infinity of numbers between any two values and their failure to associate fractional meaning to decimal positions translates to an understanding of the real number line as segmented, discontinuous, and locally finite. Nursing consequences of this conceptual disconnection were associated with students' limited knowledge relating fractions, decimals, and percents to problems in dosage calculations, use of nursing equipment, interpretation of graphs, and initial IV set up and subsequent fluid monitoring. Students' missing sense of scale, inability to make reasonable estimations, and over-reliance on cumbersome procedures enabled by the calculator, were manifestations of the conceptual disconnection that existed for them. These deficiencies resulted in students incorrectly converting 83.4 grams to 0.834 milligrams, the consequence of "moving" the decimal point in the wrong direction and the wrong number of places. Lack of meaningfully connected knowledge negatively affected efficiency and accuracy in students' work with measurements. Converting 1 pint, 0.9 fluid ounces to liters can be done by inspection by recognizing the irrelevance of the 0.9 ounces (7/1000 of a liter). Knowing one pint is approximately 0.5 liters quickly completes the problem. Yet 26% of student answers were incorrect. In one dosage calculation, a student used the calculator to support a trial and error search for a calculation that resulted in a whole number in order to avoid decimals. Students exhibited the tendency to convert metric units down the scale for the same reason.

The need for meaningful understanding of the number system is especially important in the administration of pediatric medication. Children require considerably lower dosages of important drugs and injections are sometimes measured in a small tuberculin syringe which is calibrated in hundredths of a milliliter. An accurate, reasonable dosage calculation is crucial. Pickar (1996, p. 129) in illustrating the calculation involved in a pediatric order of 0.15 mg atropine sulfate supplied in a solution of 0.4 mg per mL advises student nurses to "Be careful with the decimals. Don't be fooled into thinking 0.15 is more than 0.4." Using the nursing formula D/H x Q (the desired dose over what you have available times the volume it's in) generates the expression 0.15/0.4 x 1mL. A misstep here, of dividing the decimals on the calculator in the wrong order would indicate a truncated volume of 2.7mL be injected instead of 0.375 mL. The small 0.5 mL tuberculin syringe can not accommodate this volume, but if a nurse selects a 3-cc syringe the patient will receive 7 times the intended amount. If the 2.7 mL is misinterpreted to be 0.27 mL the tuberculin syringe can be used, but the patient would receive only 72% of the dosage prescribed. Myra and Rickie (refer to section 3.3 above) made these types of errors. Myra reversed the order of the division in calculating 6.5/8. Rickie, who depends on the calculator for all of her calculations, misread 0.6 for 0.06 on a 1-cc syringe and consequently, would have administered ten times the correct dose.

Interpreting graphical information and correctly setting up an IV drip rate are difficult tasks for students who have learned mathematics as a disconnected set of procedures. Student nurses were able to solve proportional equations without significant difficulty. In a related nursing task, however, deficient conceptual connection resulted in student failure to accurately identify a patient's pulse rate from a 6 second EKG run. They were given a 15 cm strip, scaled at 0.2 seconds per 5 mm. Students with meaningful understanding used proportionality by inspection to determine that 10 beats were displayed in 6 seconds, a pulse rate of 100 BPM. Students without adequate relational understanding did not correctly determine this rate after considerable proportional calculation. The error rate was 24% for this problem. Calculation was not the problem for students who attempted the IV drip rate problems they were presented. However, proportional reasoning beyond the calculation was problematic. The importance of relational knowledge was apparent in the appropriateness of student reasoning. Using a 15 gtt/mL (drops per milliliter) infusion set to infuse 1000 mL over 5 hours, students recorded the calculator generated value of 0.8333… drops per second. This value is arithmetically correct but is neither meaningful nor practical. To set up the IV flow by counting 0.833… drops in one second would be impossible, since the rate must be set by looking at a watch while simultaneously counting the number of drops released, and then adjusting the flow so that over a specific time interval the exact number of drops necessary to fulfill the condition of infusing 1000 mL over 5 hours will be released. In this case an appropriate setting would be 5 drops per 6 second interval. Clearly, the conceptual disconnection that results from learning mathematics as a series of rote methods has serious negative consequences in attempts to apply mathematics to nursing.

5 Summary

The study revealed that nursing students who have learned mathematics as a series of discrete rote methods struggle with basic mathematical reasoning. That struggle is propagated with ever increasing negative consequences as these students encounter the need for conceptual connection in each new context they encounter, contexts of increasing difficulty and more directly reflective of nursing problems. The same conceptual disconnection that was evident in the responses to the betweenness questions negatively affected student thinking about rational numbers and scale in each of the other sections of the protocol. Students who had difficulty with basic mathematics used unnecessarily awkward and cumbersome approaches to otherwise simple problems. The lack of relational understanding between multiple concepts inhibited flexible thought and resulted in errors in clinical problems central to nursing practice.

Students who had a meaningful understanding of mathematics gave evidence of connected knowledge. This knowledge enabled successful use of estimation to confirm calculated values. Such students were not dependent on the calculator and made effective use of numerical information in a variety of ways. Because their understanding consisted of a well developed system of conceptual connections, they could correctly interpret graphs and tables and were resourceful in solving problems in applied nursing. For these students, the completeness, correctness, and connectedness of their conceptual understanding allowed for flexibility in their mathematical thinking and confirmation of their results.

Acknowledgements

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