AN APPROACH TO COMPARISON OF CONCEPT MAPS REPRESENTED BY GRAPHS

Universidade Federal do Espírito Santo (UFES), Brasil
flavio.lamas@gmail.com, {boeres, dede, credine}@inf.ufes.br, gcarlesso@gmail.com

Abstract. We present a proposal for automation of concept maps comparison for different applications, for instance, as a support for the teacher in the task of learning assessment. Graph matching is suitable to model the comparison of objects which can be represented by graphs. The approach is based on a Combinatorial Optimization formulation for graph matching and algorithms for its resolution. This work intends to adapt it for use in the recovery of intelligent information, namely, the comparison of concept maps in representation of knowledge, as well as investigate the use of heuristic algorithms for its resolution.

1 Introduction

Concept Maps (Novak, 1998; Novak & Cañas, 2006) have been used as a tool to support the representation of knowledge. With them, it is possible to show, organize and represent knowledge about a particular subject (Araújo et al., 2002). Concept maps have been used by students to describe their understanding of a particular piece of reality. In the learning process, in examining concept maps constructed by apprentices, it is possible to identify what has been learned and the difficulties encountered, or even find concepts which are not yet understood and, therefore, need to be better dealt with. This analysis, however, can be very costly when it is necessary to review the various maps constructed by several apprentices on the same subject. The automatic identification of similarities between different maps becomes valuable in the activities of the teacher. In this task the computer can be placed as a major ally of the teacher, automating the identification of important aspects of the maps constructed.

We present a proposal for automation of concept maps comparison for different applications, for instance, as a support for the teacher in the task of learning assessment. The approach is based on a Combinatorial Optimization formulation for graph matching and algorithms for its resolution, proposed in (Boeres, 2002; Sarmento, 2005). In the next sections are presented briefly, a general proposal for concept maps comparison and its motivation (Section 2). The graph matching problem is discussed in Section 3 and an instance of the proposal using graph matching is presented in Section 4. Section 5 shows tools to construct data input for this problem, the algorithm used for its resolution and computational results. Conclusions and future work are in Section 6.

2 Comparing concept maps

Concept Maps (Novak & Cañas, 2006) can be used to support knowledge representation and organization. According to Piaget, a concept results from a change in a scheme of action, in a process of endless juxtaposition of attributes by successive adjustments caused by disequilibrium (or imbalances) in the systems of signification of the subject. Thus, the words we put in the boxes of the maps (usually a noun) are not necessarily, in the view of the subject, the concepts. Even though such words may represent them, they are in fact delimited by the relationships created in the exercise of allocating meanings that are only achieved by the interaction of the subject with objects, in certain contexts. Therefore, it is opposed to the vision of a sequential and hierarchical mental structure built during a learning process (Fagundes, Dutra & Cañas, 2004). Safayeni also advocates that the cyclical concept maps, that is, not hierarchical maps, can be more effective for a more dynamic knowledge representation, allowing a greater possibility of a concept map configurations, both in its topology as in its type of connecting sentences (Safayeni et al, 2003). The addition of these two concepts is the notion of concept maps adopted in this article.

Concept maps are popular today and they are used to support different activities where knowledge needs to be organized and represented (Gava et al., 2003), notably in education (Dutra et al., 2004). In human activities we are taken by the curiosity to know their similarities and/or differences, and also to compare them. But, in addition to satisfy our curiosity, the comparison of concept maps may have other uses, as we can see in the following examples:

• A teacher asks his students to build, individually, concept maps on a particular issue and compare them to identify existing differences and similarities;
A knowledge engineer calls for different experts to build concept maps describing the knowledge they have on a particular subject, such as "Mars". The comparison between them will allow to obtain a more precise description of the subject in question.

Different texts can be described by means of concept maps. The comparison between these maps will allow knowing the degree of similarity between the different texts.

We can mention a simpler problem: given two concept maps, CM1 and CM2, what are the similarities between them? The treatment of the problem would be considerably simplified if the two maps were constructed using the same vocabulary and naming the concepts and relationships in a non-ambiguous way. We could make a comparison of the two maps and count the number of overlapping concepts, the number of concepts that appear in CM1 and do not appear in CM2, and vice-versa. Similarly, we can proceed with the relationships. Yet some major questions now arise: (1) the position of the concepts and relations in the figures used to describe the maps (layout), and (2) lack of uniform nomenclature to define the concepts and relationships presented in the maps.

2.1 Using graphs to compare concept maps

In addition to its pictorial representation, a concept map has an underlying structure of connections between their concepts. Because of this, equivalence between concept maps and a mathematical structure called graph can be established. It is important to note that, by establishing this equivalence, the existing knowledge on graphs can be applied to compare concept maps. Our strategy is to address the problem of comparing concept maps to a known problem in graph theory literature: the graph matching problem. In Figure 1, two different graphical representations for a concept map with the same concept relationships are presented. It can be observed that concepts of the same nature are related in the same way in both, even if it is not evident from the graphical representations. For instance, the relations *whale may be animal*, from one of the maps and *whale may be creature*, of the other, have similar meaning. These similarities can be also observed for the concepts and the other relationships established in both maps. Similarly we can find maps where the description of the relationships between two concepts is made by different sentences. It is also possible that a map has less relationships that another map.

**Figure 1.** Two different graphical representations for the same concept map.

A graph $G = (V, E)$ is defined as a pair of a set of vertices (or nodes) $V$ and a set of edges $E$. The edges represent relationships between the vertices. They can be oriented or not, depending on the nature of relations represented. An attributed graph can have labels representing attributes to its nodes and edges, depending on the context of the problem modeled (Berge, 1983).

A concept map can be defined as an attributed graph $G = (V, E)$ where the $V$ set contains the nodes labeled by concepts and the $E$ set contains the edges that represent every relationship between two concepts. The edges attributes can be words or phrases used to describe the relations between concepts. And, as the concept maps discussed in this work can be constructed with free vocabulary, different map constructors can use different words or phrases for a same concept and/or relationship. For example, in a statement talking about housing problems, a user can choose the word "house" and another, the word "dwelling" to talk about the same concept (housing). Thus, given two concept maps CM1 and CM2, the basic problem is to find a concept $e'$ or a relation $r'$ in CM2 that most closely matches a concept $e$ or a relation $r$ in CM1. This problem will be treated in this work using graph matching (Section 3).
The approach proposed in this work considers the comparison of two concept maps represented by attributed graphs. The maps comparison is performed using graph matching. For this, a semantic comparator is used to calculate the similarities among the concepts and relations, represented as attributes of both graphs. Thus, a solution to the graph matching problem represents an association between the concept maps compared. The whole scheme of this approach is presented in Figure 2.

Figure 2. Schematic comparison of two maps.

3 The Graph Matching Problem

In the literature of graph theory one finds the problem of Graph and Subgraph Isomorphism formulated as a decision problem (Berge, 1983), that is, given two graphs, it is intended to identify the complete structure of a graph, or just a part of it, in the other graph. Motivated by image recognition applications, the graph matching problem is proposed in (Sarmento, 2005) as a combinatorial optimization formulation for the graph isomorphism problem (GIP). The GIP goal is to identify similarities between attributed graphs of the same size, considering their structures and attributes associated with their nodes and edges. Details of similarity, node to node and edge to edge of the two graphs, must be provided. This information can be calculated through metrics that consider cognitive data (the attributes associated with the graphs) and they must be stored in two matrices of similarity, one of them between nodes and the other between edges of the graphs compared. The formulation of the GIP as defined in (Sarmento, 2005) is reproduced below.

Let $G_1 = (N_1, E_1)$ and $G_2 = (N_2, E_2)$ be the compared graphs, with $|N_1| = |N_2|$ and $|E_1| = |E_2|$, and be still matrices with dimensions $|N_1| \times |N_2|$ and $|E_1| \times |E_2|$ of values in the range $[0, 1]$, obtained from the graphs attributes, that represent respectively, the similarities between the nodes and edges of the two graphs. A solution to the GIP is a correlation $X$, between the nodes of $G_1$ and $G_2$, which maximizes the function

$$f(X) = \frac{\alpha}{|N_1| \cdot |N_2|} f^x + \frac{(1-\alpha)}{|E_1| \cdot |E_2|} f^a$$

with $f^a = \sum_{(i,j) \in E_1} \sum_{(i',j') \in E_2} (1 - \max \{x_y \cdot x'_{y'}, x_{y'} \cdot x'_y - s^a((i,i'),(j,j'))\})$

where $\alpha$ is a parameter used to weight each term of $f$. The first term on the right side of $f$ represents the average contribution of the graph nodes associations for the matching, while the second term represents the average contribution of the graph edges associations. The value $s^x(i, j)$ (respectively $s^x(i', j')$) is the similarity calculated from the attributes of the nodes $i \in N_1$ and $j \in N_2$ (respectively the edges $(i,i') \in E_1$ and $(j,j') \in E_2$). Restrictions have also been defined and imposed on the space of solutions in order to improve the search process of the best solution. The definition of these restrictions was based on the identification of characteristics of the problem treated and can be found in (Sarmento, 2005). A GIP feasible solution must satisfy the restrictions set imposed to the problem. The GIP formulation described in this section is used in this work to model the concept maps comparison problem.
4 Comparing concept maps using GIP

In this work, we address the problem of comparing concept maps to the GIP, described in Section 3. So, in this case, we consider that concept maps are treated as the graphs $G_1$ and $G_2$ and aim to find a solution to the GIP with the best value for the function $f$. In other words, we aim to find the solution that best represents the similarities of the two maps. We denote this problem as CMGIP. To solve the CMGIP, node and edge similarity matrices must be provided for the maps. In this approach, these matrices can be built from semantic algorithms as stemming and disambiguation algorithms.

The GIP, in its classical version, is NP (Fortin, 1996; Arvind, 2002). For this reason, approximate and exact algorithms are proposed for its solution, in several applications. As an example, in the literature of scene recognition based on this problem, there exists for its resolution, heuristic algorithms (genetic algorithms (Cross, 1997), neural networks (Buchanan & Shortliffe, 1984) and GRASP (Boeres, 2002; Sarmento, 2005)), probabilistic methods (Bengoetxea, 2002) as well as exact algorithms based on the branch-and-bound technique (Wong et al., 1990). In the approach proposed in this work, several algorithms for GIP resolution are suitable. We have adapted the heuristic GRASP proposed in (Boeres, 2002; Sarmento, 2005) for the CMGIP.

An instance of the approach scheme presented in Figure 2, considering the similarity matrices creation and algorithm for the CMGIP resolution, is showed in Figure 3. In this scheme, concepts maps CM1 and CM2 are represented as graphs $G_1$ and $G_2$ and their attributes (concepts and relations) are extracted and compared by a semantic comparator to construct the node and edge similarity matrices, needed to solve the CMGIP. Finally, a GIP algorithm is chosen to obtain a solution to the problem and have the concept maps compared, identifying their similarities.

5 An instance of CMGIP

In order to evaluate the contributions of the approach proposed to the comparison of concept maps, an instance of CMGIP is described in this section. For this, in the next two subsections, the matrices creation and a CMGIP resolution algorithm are described further. Subsection 5.3 presents the results of the algorithm implementation for a specific set of concept maps.

5.1 Construction of similarity matrices from concepts and relations of two maps

The comparison of maps via CMGIP assumes the use of similarity matrices whose values are calculated from attributes associated to the graphs compared. In this approach, these attributes are concepts (for the nodes) and words or phrases, meaning concept relations (for the edges). For the matrices construction, it is necessary to quantify these similarities by means of numeric values. For this purpose, natural language processing algorithms were chosen: stemming algorithms (Rijsbergen, 1980), disambiguation (Gerrig & Lesk, 1990), and synonyms tree search algorithms created from WordNet (Fellbaum, 1998). These algorithms were implemented in this work from available versions for use on the Internet. However, they are restricted to the comparison of English language strings. Given two strings, these algorithms generate a numerical value (percentage), indicating how
much two strings are semantically similar. In order to validate the use of these algorithms, the Microsoft Research Paraphrase Corpus tool (Quirk et al., 2004) was used to evaluate the quality of the comparisons made. It provides several pairs of sentences in English and their percentage of similarity, defined according to the assessment of two human judges. From the 5801 tests made we obtained 3909 hits, or 67% of fidelity in relation to the assessment made by the judges.

5.2 The algorithm GRASP used to solve the CMGIP (GCMGIP)

The metaheuristic GRASP (Greedy Randomized Adaptive Search Procedure) (Feo & Resende, 1995) is an improvement algorithm that generates good solutions (not necessarily the optimal solution) to a problem in a fairly processing time. This algorithm has been widely used in the resolution of combinatorial problems. It is an iterative algorithm with each iteration consisting of two phases: (1) construction of a feasible solution to the problem, and (2) its use as initial solution to a local search procedure. The solutions obtained at each GRASP iteration represent the exploitation of the search space by a local search from multiple starting points. The best among all the solutions obtained is the response of the algorithm.

The GRASP algorithm used in this work is that presented in (Sarmento, 2005): at each iteration of the algorithm, a solution is gradually built by elements chosen among candidates that do not violate the feasibility criteria established in GIP by the restrictions set defined. Then, this solution is used as a starting point for local search, performed in a neighborhood of solutions derived from the constructed one, from exchanges of associations established between nodes of the graphs compared. The parameters of the algorithm are: the graphs, the maximum number of iterations and an initial seed for the random number generator, used for the random selection of the elements that will compose the solution built. The algorithm ends with the best of the solutions obtained, after the execution of the maximum number of iterations. The GRASP algorithm pseudo-code is presented in Figure 4.

```
Input: G1 = (N1,E1), G2 = (N2,E2), Seed, MaxIter
i = 1
While i < MaxIter do
    Solution = Construction-Procedure (Seed)
    Solution = LocalSearch (Solution)
    i = i + 1
    UpdateSolution(Solution, BestSolution)
End-While
Output: BestSolution
```

Figure 4. The GRASP algorithm

The input maps are easily represented by attributed graphs of the same size \((G_1 \text{ and } G_2)\) and their similarity matrices are created using the algorithms mentioned in Section 5.1. Considering the graphs and matrices, the GCMGIP algorithm construct an initial feasible solution (Construction Procedure) and starting on it, performs a local search in the problem solution space (Local Search), guided by the GIP objective function presented in Section 3. It returns the best solution found. As the GCMGIP is not an exact algorithm, the best solution obtained can be not the optimal one.

Maps of different sizes can be easily adapted to this algorithm by completing the lower graph with edges and nodes with null similarity values, so that their contribution are not considered in the values of the objective function, calculated for the solutions in the algorithm.

5.3 Experimental Results

Whereas we are working with heuristic algorithm, and as it is hard to estimate its complexity, we choose to make a preliminary performance evaluation in the resolution of specific situations. For the computational tests, nine pairs of attributed graphs were built from nine concept maps acquired in the Public CmapServers (Cañas et al. 2004). Four groups of these maps were organized for the algorithm executions: (1) The M0 group consists of pairs of identical maps, just to validate the algorithm, (2) The M1 group consists of pairs of maps with identical graphical representations but with some of the concepts replaced by close meanings or synonyms, (3) the M2 group consists of pairs of maps with different graphical representations but with similar concepts, and (4) the M3 group, consists on the union of M1 and M2 groups, with modifications imposed both on the graphical representations and on the concepts attributes of the maps. Even with the modifications imposed on the graphs, they should be identified as identical ones, because all pairs of graphs are isomorphic. For instance, Figure 4 shows an example of the M3 group. The underlying graphs of these concept maps are easily determined: each concept is defined to a graph node and each relationship between two concepts is defined to a graph edge. For
instance, the concept relationship *exempli gratia*, in the left concept map of Figure 4 is converted to three different edges with the same attribute (*exempli gratia*), in its underlying graph.

**Figure 4.** An instance of the M3 group.

Ten executions of the GCMGIP algorithm were carried out for each pair of maps of the groups M0, M1, M2 and M3. All tests were performed on an Athlon XP 2000+ computer with 768MB of RAM, operating system Windows XP SP2 and running code compiled into C# in Visual Studio 2005. Tables 1, 2 and 3 show the results obtained respectively for the groups of examples M1, M2, and M3. In each table, the first column indicates the instance reference. In the second column, the values |V|, |E|, |V|´ and |E|´ represent, respectively, the number of nodes and edges of the graphs G1 and G2 in each instance and the number of changes imposed on concepts and on their relationships. The third column indicates the average similarity percentage (from the ten executions) of the compared graphs and the execution time in seconds, obtained by the GCMGIP algorithm.

### Table 1: Experimental Results (group M1).

<table>
<thead>
<tr>
<th>Instance</th>
<th>G1 and G2 vertex and edge sets sizes and number of changes</th>
<th>GCMGIP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>V</td>
</tr>
<tr>
<td>1</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>2</td>
<td>17</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>16</td>
</tr>
<tr>
<td>4</td>
<td>27</td>
<td>32</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>30</td>
<td>41</td>
</tr>
<tr>
<td>7</td>
<td>28</td>
<td>35</td>
</tr>
<tr>
<td>8</td>
<td>14</td>
<td>15</td>
</tr>
<tr>
<td>9</td>
<td>12</td>
<td>14</td>
</tr>
</tbody>
</table>

### Table 2: Experimental Results (group M2).

<table>
<thead>
<tr>
<th>Instance</th>
<th>G1 and G2 vertex and edge sets sizes and number of changes</th>
<th>GCMGIP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>V</td>
</tr>
<tr>
<td>1</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>2</td>
<td>17</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>16</td>
</tr>
<tr>
<td>4</td>
<td>27</td>
<td>32</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>30</td>
<td>41</td>
</tr>
<tr>
<td>7</td>
<td>28</td>
<td>35</td>
</tr>
<tr>
<td>8</td>
<td>14</td>
<td>15</td>
</tr>
<tr>
<td>9</td>
<td>12</td>
<td>14</td>
</tr>
</tbody>
</table>
From the information in the tables, it can be observed that the GCMGIP algorithm succeed to identify, in average, 89.77% of similarities between the maps compared in a very short execution time for all instances. As the concepts and their relationships are unchanged in the M2 group, the GCMGIP presented its best performance in this case, succeeding to recognize completely all but one instances. Also, the algorithm identified with 100% of similarity, all instances of the M0 group, whose pair of maps are completely identical.

6 Conclusions and future work

In this work, concept maps are described as attributed graphs and its comparison was performed using graph matching, more specifically, graph isomorphism. For its solution, a heuristic algorithm was used to automatically compare the maps and compute their similarities. The experimental results obtained so far indicate that the use of automated techniques for the comparison of concept maps is suitable for several applications. For instance, it can provide an efficient way of monitoring and evaluation of procedures for learning as well as the classification of documents represented by concept maps.

This proposal is generic and can be applied to concept maps represented in any language. But, as it needs a words comparator, a preliminary instantiation of this proposal was implemented to compare concept maps described in the English language. The adaptation to any other language depends only on the replacement of the module for the comparison of words.

Future works will mainly consist of the validation and subsequent implementation of the described techniques in real situations of learning, applying them to concept maps, eventually of different sizes, constructed by Computer Science students. Furthermore, we intend to use concept mapping for summarizing discussions in thematic forums and also for representing and comparing ontological concepts.

7 Acknowledgements

We thank Markku Kari for his assistance during English translation of this manuscript.

References


