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CONCEPT MAPPING IN MATHEMATICS: TOOLS FOR THE DEVELOPMENT OF COGNITIVE AND NON-COGNITIVE ELEMENTS

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Abstract. The present work deals with a teaching experience with students beginning their formation in engineering at a university level. It deals with a non traditional course of Calculus using concept mapping as a teaching tool. Considering learning by the students to be a development of both cognitive aspects as well as non-cognitive aspects, concept mapping was integrated into the course in order to develop different components of logical reasoning and spatial orientation, as well as to develop a component to the non-cognitive sphere: the self-concept. The teaching experience described represents a phase of experimentation forming part of research into the teaching of mathematics. It is an attempt to show that in the framework of the cognitive paradigm of education mathematics and teaching strategies with concept mapping as a tool can be considered as a means to the development of thought and affectivity, that is to say, for the cognitive and non-cognitive.

1 Introduction

At the different teaching levels, the learning process in mathematics, or any other content, implies the development of cognitive and non-cognitive elements, that is to say, elements of thought and elements of affectivity. Polya (1972) asserts that it is not right to consider the solution to a problem to be merely an intellectual question. The author explains that in the activities of mathematics and science, the emotions play an important part. To resist years of work and eventual failures, Polya (1972) pointed out certain intellectual development is necessary but also a great strength of will. In the field of learning of mathematics the development of intellectual capacity is important, but also affective aspects can motivate the student for the activities and tasks. These basic elements of affectivity can be stimulated in the classroom during the teaching and learning processes.

During the study of mathematics cognitive elements can be developed such as logical reasoning and spatial orientation. In other words, the basic stages of thought can be developed such as induction, deduction, placing, locating and graphic expression, among others. This is what can be appreciated in the cognitive paradigm of education. From this perspective the teaching of mathematics can be understood as one way, or particular ways, of proceeding in the classroom in order to contribute towards learning, understood as the development of various cognitive capacities.

But during the learning experience in the classroom, what is known as the self-concept, considered as a non-cognitive element belonging to the field of motivation and affectivity, can be developed. Motivation achievable in the class room coincides favourably with affective factors of the student's personality, which in turn helps the learning process. The self-concept is considered to be an affective factor which should not be forgotten at any teaching level. This affective factor (non-cognitive) is not to be found in isolation; as Nuñez (1998) states, it is an element that influences cognitive strategies.

A good range of strategies in the classroom leads to teaching that favours learning of quality. This can be translated, according to Goleman (1995), as a development of elements both intellectual (cognitive elements) as non-intellectual elements (affective elements such as self-concept).

2 Concept mapping in the learning of mathematics

There are different ways of using concept mapping as a tool to support the learning of mathematics in the classroom. The following describes a particular way of applying concept mapping, which can be summed up as follows:

- Concept mapping elaborated by the teacher serves in the main as a guide to reinforce the development of thought elements that are implicit in mathematical content (cognitive elements).

- Concept mapping developed by the students as classroom activities, supervised by the teacher, serve as reinforcement in the development of affective factors such as self-concept (non-cognitive elements).

In this same section methodological aspects will also be presented as well as results obtained during teaching experiences with beginning students in engineering.

2.1 Concept mapping for the development of cognitive elements.

One must not forget certain ideas about concept mapping and its applications in the teaching context. It is important to mention that concept mapping (Novak, 1998) plays a key role in the classroom in the representation of knowledge. Concept mapping is a useful aid for the teacher since it is a good tool in how to organize the knowledge and show it (Novak, 1998), but it also helps the students in their efforts by being able to learn the contents in a constructive and meaningful way, quality learning (not by rote). In this sense the use of concept mapping in the classroom together with a group of teaching strategies allows the development of capacities and cognitive skills in the students (Román, 1988), that is to say, a development of thought and a development of elements of cognition. This can be translated into an adequate storage of the material in the student's cognitive structure to be available when needed (Ausubel, 1988). One can say that the performance of the teacher, as a mediator in the classroom, guided by concept mapping, leads to cognitive intervention.

How does the teacher act guided by concept mapping in order to develop the thought processes of the students? The teacher has to prepare his own concept mapping with the contents of the mathematics where concepts appear with different levels of generality, with graphics or images associated with the concepts and concrete examples included. The classes are not just a question of presenting the students with the concept mapping created by the teacher. His mapping is only a guide to teaching. One has to present the material in the mapping little by little trying to go from the particular to the general, naturally amplifying and enriching the information. In other words, move from the information in the lower part of the mapping to that in the upper part. The idea is to present particular information accessible to the students' level in order to be appreciated later with the support of images and/or graphic representations and arrive at the presentation of concepts. It is very important to take into consideration that graphics or images contribute to the visualization of the concepts, to the learning of mathematics (De Guzman, 1996). In addition, moving from the particular to the general promotes the realization of inductive thought processes, which means a development of thought or cognitive intervention (Feuerstein, 1995).

The intellectual work carried out by the teacher when creating his maps implies the consideration of the teaching-learning process, in other words, finding out how the student learns in order thereby to design his teaching. The stages of perception, representation and conceptualization are considered to be basic stages in learning (Román, 1988). The concept mapping is a guide to favour these stages in the classroom. By promoting perception, representation and conceptualization, in this order, inductive thought processes are put into practice.

For the teaching of mathematics concept mapping prepared by the teacher must take into consideration the learning profiles of the students (Krutetskii, 1976). They must contain both visual elements (geometrics) and analytical, to help those students who have a strong inclination towards the visual aspects of mathematics and to attend those inclined to analytical reasoning.

To sum up, the teacher must present both facts and concepts. As a particular interpretation of Piaget ideas, it is considered that by contrasting facts with concepts and concepts with facts, inductive and deductive processes are carried out, contributing to constructive learning (Piaget, 1979). The teacher must present the concepts beginning from the lower levels and leading to the higher levels of generality. As Ausubel explains, the inductive and deductive thought processes are empowered when the information is disposed respecting the conceptual hierarchies, achieving subordinated and supra-ordered learning, moving from the particular to the general and vice versa (Ausubel, 1976). The teacher must also initiate from a stage where the student can handle the information in order to arrive at abstraction, passing through the visuals supports. From the point of view of Bruner and the theory of learning by discovery, moving from an inactive system to a symbolic system allows the development of inductive processes (Bruner, 1988).

Figures 1, 2 and 3 in the following pages show some examples of concept mapping constructed and integrated into the process of the teaching and learning of mathematics during a course of calculus. It is important to point out the teaching and learning of mathematics not only centres on concepts, but it must also pay much attention to exercises and problems as well as the development of the abilities to resolve them. For this reason, concept mapping not only covers concepts, but it is also important to show aspects of the procedure for problem solving.

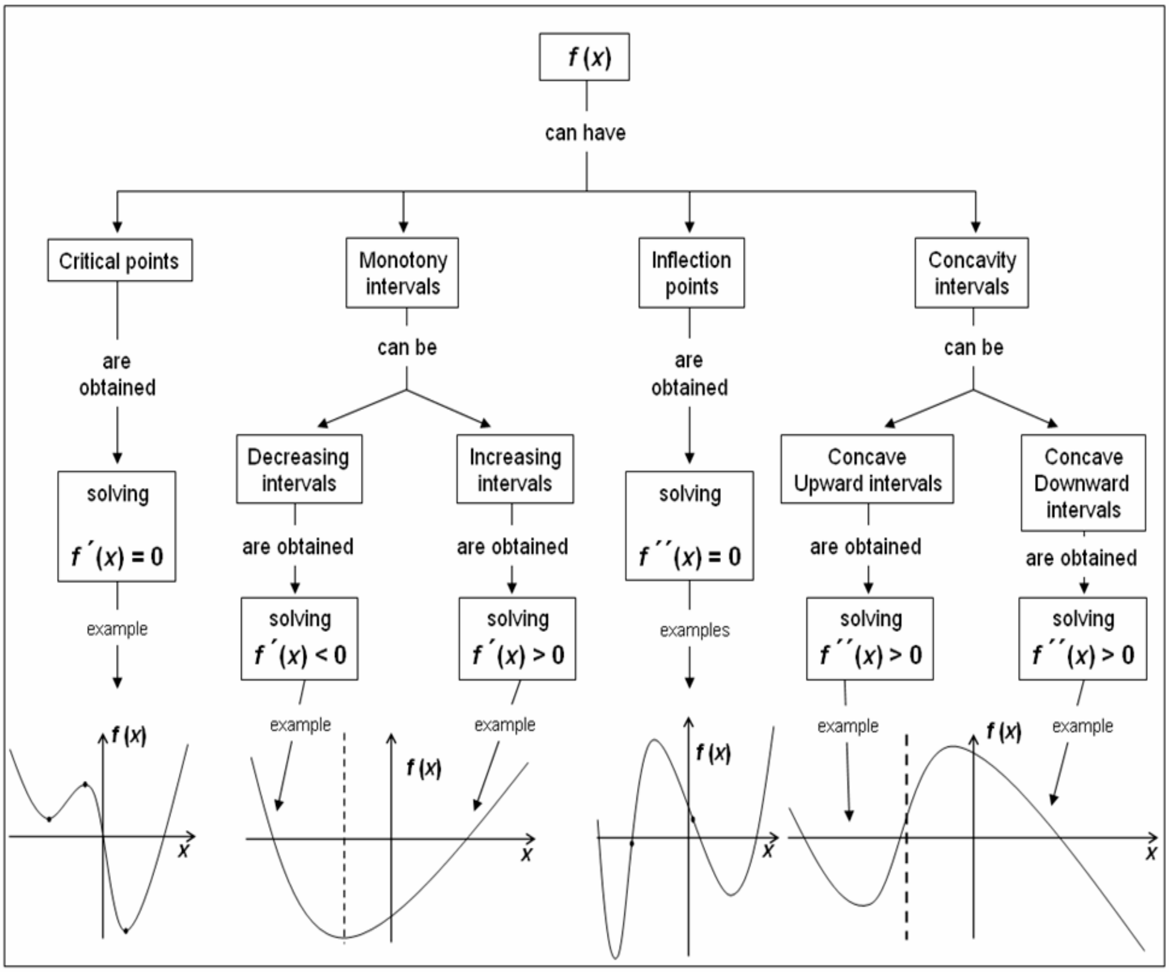

Figure 1. Concept mapping: "Elements of a function".

2.2 Concept mapping for the development of non-cognitive elements.

Teaching practice has not paid enough attention to the emotional impact of the learning of mathematics. In fact, most of the literature dealing with the subject of research into the learning of mathematics only takes into consideration the cognitive aspects, forgetting the non-cognitive elements (affectivity), mainly because the study of mathematics is prejudged to be merely intellectual or cognitive.

On the other hand, various authors (Núñes, 1998) point out that the learning of mathematics in the atmosphere of the classroom has innumerable connotations, that mainly refer to anguish, suspicion, boredom, and what is worse, personal beliefs about the lack of cognitive competence, (believing one is no good at mathematics). It is a question of connotations that belong to the ambience of motivation and affectivity. From that point of view, one can insist that the learning of mathematics is not only a question of cognitive capacity.

Methodological strategies for the teaching and learning of mathematics must be designed as an interactive system in which cognitive capacities and feelings combine. Gómez (2003) points out that in the degree that such aspects become integrated, relationship and pedagogical communication established between the teacher and the student, represent a factor that facilitates the acquisition of mathematical concepts and abilities. But also, this interactive system favours the development of affective elements in the students such as self-concept. In the context of the classroom the self-concept can be understood as the interaction of the student with the others and also as the influence he receives during coexistence. Self-concept is a dynamic and active construct of the student's physical, cognitive and emotional history.

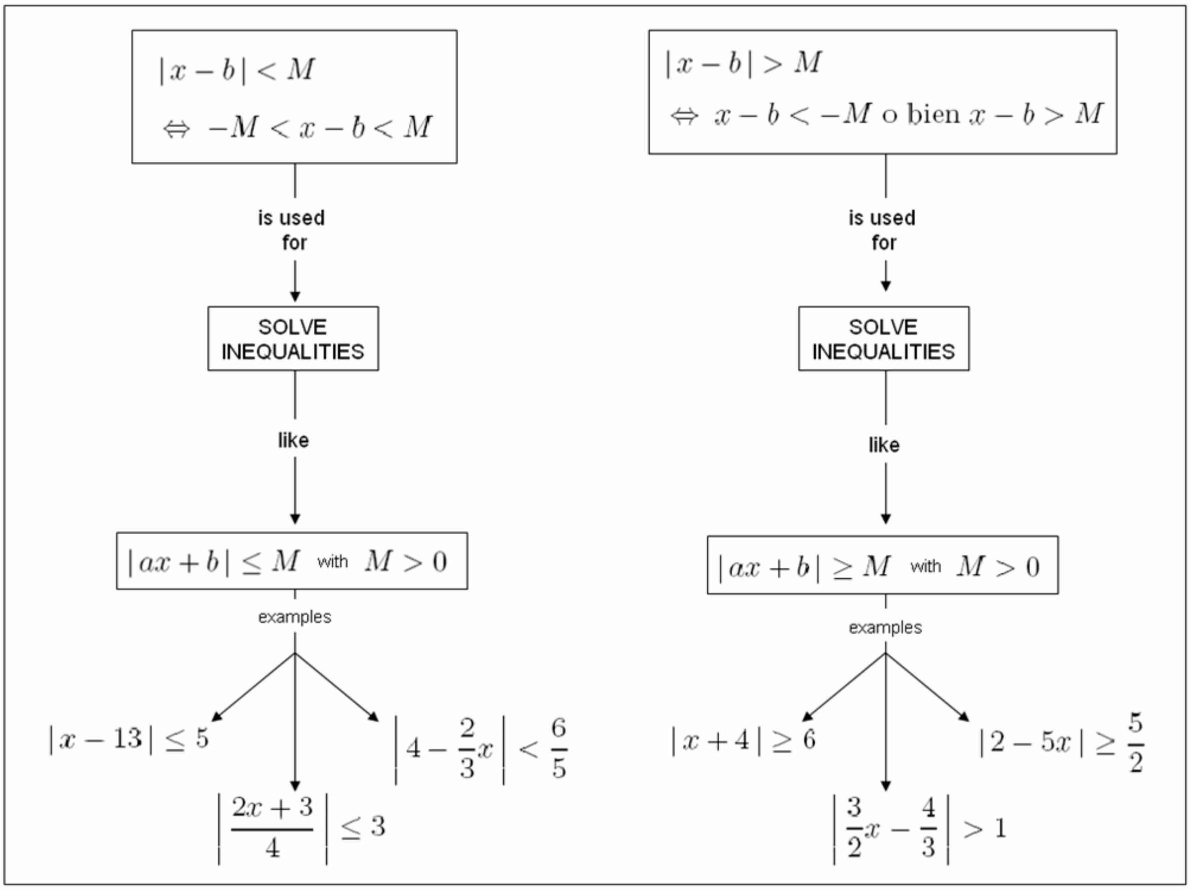

Figure 2. Concept mapping: "Solution of certain inequalities".

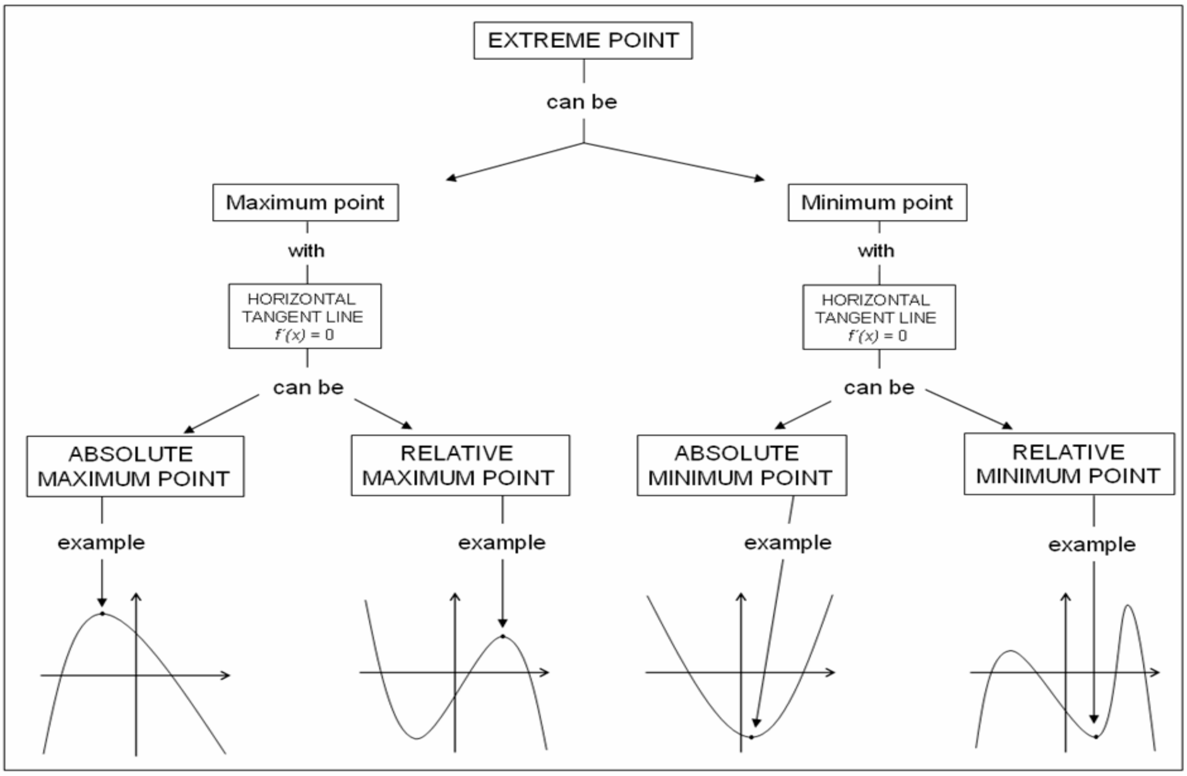

Figure 3. Concept mapping: *"Extreme point"*.

Concept mapping integrated into the teaching activities influences learning, understanding this as the development of cognitive and non-cognitive elements. Especially the concept mapping the students can develop as activities in the classroom, under the supervision of the teacher, helps mainly in the development of affective factors, such as self-concept.

The students' activities, collaborating with each other and the teacher, such as the preparation of concept mapping and the development of exercises or problems, represent a significant cognitive act and have an effect upon motivational aspects. A significant cognitive act has an effect upon self-concept (González, 1987).

Taking up some of Rogers' ideas (1990), the teacher's role as the facilitator of knowledge can create cognitive conditions and ambiences favouring the development and construction of learning. Rogers (1990) describes the interaction between the teacher and students as a horizontal relationship and not hierarchical, in that the communication is conceived as part of the pedagogical relationship, and in that the intellectual, emotional and affective experiences are integrated. The basic idea is the proper development of the self-concept should interact significantly with the motivational dimension and cognitive effort, which reflects upon the optimum and significant learning processes.

2.3 Educational experience with concept mapping for the study of mathematics

It is obvious one can state that the subject of mathematics is of great importance, among other reasons for its application in science, in technology and contexts in daily life. But in addition, as with other subjects, mathematics is of great educational value at all levels, since it allows for the development of thought. In addition, at all levels of the educational systems problems appear in regard to the teaching and learning of this subject. Perhaps this educational truth has increased the interest to undertake from different points of view studies that offer an alternative or guide to teaching and learning.

The following is a synthetic description of the most important aspects of an educational experiment in which it was realized concept mapping represents a teaching tool that is very useful within the process of the teaching and learning of mathematics. In particular it was observed that concept mapping are an important tool for the development of both cognitive and non-cognitive elements for students beginning their formation in mathematics in the field of engineering.

In the Autonomous Metropolitan University (UAM) of Mexico City two groups of students were selected: a control group and an experimental group, each with 30 students. Both groups were formed of students starting their initial formation in engineering. The control group has an ordinary course (traditional course) in the subject "Calculus I" and the experimental group had the same course following a process of teaching-learning supported by concept mapping. Both groups had a daily session for three months, taking into consideration the programming of the Autonomous Metropolitan University.

One of the relevant characteristics of this teaching research was that the teacher of the experimental group created a collection of concept mapping for the contents of the course "Calculus I" and used them as a guide for his teaching. The maps orientated him upon the way to present the information in order to promote the perception of information accessible to the students' intellect. The maps also contained graphic and visual information in order to support the mental representation and understanding of concepts. The teacher of the experimental group concentrated on putting into practice two elements of thought characteristics of logical reasoning: induction and deduction. Also put into practice were elements of thought proper to the capacity of spatial orientation: situating and locating in the line, in the Cartesian plane and in space.

Also during the sessions of the experimental group sub-groups were organized for team work under the very careful supervision of the teacher in each of the activities, mainly in the elaboration of concept mapping by the students. At each moment there was a horizontal relationship between the students and the teacher, promoting inductive and deductive thought processes, taking care of the appropriate level of exercises for each student, taking care in each sub-group of the correct construction of concept mapping, supporting the students with graphic and visual elements for the successful realization of exercises and problems. At every moment the doubts expressed by the students were resolved and an atmosphere of cordiality, confidence and respect among all of the participants of the experimental group was promoted.

To find out in what measure concept mapping used in the classroom contribute to the development of cognitive and non-cognitive elements, the following was carried out: for both the control group and the experimental group pre-tests were applied that measure Numerical Reasoning, Abstract Reasoning and Spatial Relationships (differential aptitude Test Dat-5). The test AF was also applied: Self-concept form 5 (García, F., & Musitu, G., 2001). Before initiating the training previous analyses were carried out (Student's **t**) which could check the two groups, both experimental and control group, were homogeneous in regard to Numerical Reasoning, Abstract Reasoning, Spatial Relationships and Self-concept, from the beginning. At the end of the course (training) post-tests were applied of the same instruments to all the students of both groups.

In this research the following hypotheses were maintained: If a group of university students (experimental group) receive a course of the subject "Calculus I" following a Teaching-Learning process backed by concept mapping and the results are compared with those of another group (control group) of similar characteristics who receive the same subject in a traditional form:

1. A noticeably superior increase can be observed in Numerical Reasoning – measured by Test "Dat-5" – in the members of the experimental group in regard to those of the control group.
2. A noticeably superior increase can be observed in Abstract Reasoning — measured by Test "Dat-5" – in the members of the experimental group in regard to those of the control group.
3. A noticeably superior increase can be observed in Spatial Relationships - measured by Test "Dat-5" – in the members of the experimental group in regard to those of the control group.
4. A noticeably superior increase can be observed in Self-concept – measured by Test "AF5: Self-concept form 5"—in the members of the experimental group in regard to those of the control group.

2.4 Results

To evaluate the differences in the results obtained between the pre-test phase and the post-test phase, between the experimental and control groups, an analysis was carried out of the results obtained, to which the following tests were applied: Student´s **t** (Parametric Test) and Assigned Ranks of Wilcoxon (Non- parametric Test). The results obtained permitted the verification of the truth of the hypotheses of the research. After processing the information from the Dat-5 measuring Numerical Reasoning for the two groups, statistically significant differences could be observed in the experimental group with a level of confidence of 99% between the pre and post test. Processing the information from Dat-5 measuring Abstract Reasoning and Spatial Relationships revealed significantly different statistics with a level of confidence of 99% between pre and post tests. Also, the processing of AF5 measuring Self-concept revealed significantly different statistics with a level of confidence of 95% between pre and post tests.

This affirms that there was a significant evolution in Numerical Reasoning, abstract Reasoning, Spatial Relationships and Self-concept in the experimental group. This increase is mainly explained by effect obtained by including in the teaching-learning process in a course of calculus, concept mapping as tools for the development of cognitive and non-cognitive elements

During the teaching experience there was an opportunity to obtain information from the students of a qualitative kind. Among the most important ideas collected are the following:

- Interest and tranquility are feelings expressed by the majority of students during the teacher's presentations and during the activities organized in the classroom. The students considered appropriate the presentation of specific information (examples), accessible to their intellect, supporting the introduction of concepts. They say that in this way they receive many elements to comprehend concepts and procedures to resolve problems. They say they feel comfortable and not stressed as when they do not follow the teacher.
- They rated favourably all the information received by electronic means (computer and projector) projected in images or graphic representations as an aid to comprehending theoretical aspects. Some students assured they had assimilated ideas and resolved successfully the exercises by relying on mental or graphic representations, referring to the well know phrase: in picture is worth a thousand words.
- The concept mapping prepared by the students themselves are important. They consider them back up documents for their personal study. The construction of the maps is attractive for them and it is rewarding to know the different ways to understand the concepts. They feel free to express their ideas without being afraid to be prejudged or disqualified. The students take the initiative trying to resolve exercises although they may come up against obstacles.
- The students consider it is important to know the concept mapping designed by the teacher. They feel this activity motivates them to construct their own maps with the contents of the course, thus developing their thought processes. They worry about comparing their own maps with those of the teacher.

- The students mention one of the important aspects of the course: they refer to the fact they have understood the theory and not memorized it. They consider they have made substantial learning advances. Some students state that certain concepts and procedures to resolve problems learnt in previous courses they still recall but without remembering the content: The why and wherefore. They feel the capacity to face up to calculus exercises that previously meant an unbeatable challenge.

3 Conclusions

One of the important aspects of this research is to be able to appreciate fully how concept mapping represent a useful tool to develop both elements of thought and elements of affectivity. One realizes that in the process of teaching and learning of contents proper to the field of the exact sciences, such as mathematics, the non-cognitive aspects are involved and the development of them is also important. It is very motivating for learning when the student experiences changes in regard to the concept he has of himself. The learning process is favourably affected when the students discover they have the capacity to solve mathematical problems that are apparently complicated. Naturally the research requires information of various stages of experimentation to continue constructing and refining statements that in their turn form the theoretical framework.

There are many ways to proceed in the classroom to achieve the learning of contents at the different educational levels. Using concept mapping as a guide for the teacher is a particular way to act. The teacher, by initiating the course with particular information and the support of graphic representations and images can get the concepts to generate in the classroom an atmosphere that favours mental activities in the students. They are not just passive observers, but from the beginning perceive, represent and conceptualize. Setting off the inductive and deductive processes implicit in mathematics develops intellectual performance: thought. By active participation in the classroom understanding the tasks carried out helps modify their attitude toward the contents and self-concept.

To carry out a teaching experience such as described here is a challenge. The activities to be designed must have a foundation in different ways to learn the contents. One may think that concept mapping are merely a different way to arrange or organize the contents. However, one must not lose sight of the fact that behind the concept mapping there is a theoretical body that sustains the idea. To include concept mapping as a teaching tool used by the teacher requires deep reflection on his part in regard to their immense potential. In this way teaching takes on special different aspects in relation to the traditional methods of proceeding in the classroom. The university requires new research into the processes of teaching and learning. Research to enrich pedagogical knowledge involving: students, teachers, contents and technology.

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