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PICTORIAL SCAFFOLDING IN THE CONSTRUCTION OF SCHEMA OF CONCEPTS

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Abstract. The learning obstacles experienced by community college students in the Bronx in mathematics classes have been reported in several issues of the online Teaching-Research Journal on Line. (MTRJoL, 2006-2008) Over the course of the teaching research studies utilizing the Teaching-Research/NYC model, it was found that students absence of assured success is caused by absence of independence of learning, difficulty getting started, absence of procedural skills such as multiplication facts, all of which contribute to weak problem-solving skills which in turn create obstacles in students' capability to adopt a problem-solving approach to learning the very basic mathematics in question. The Discovery-based Instructional Sequences utilized in our classrooms are designed from the point of view of facilitating the construction of a schema among students, with a steady emphasis on the Zone of Proximal development of Vygotsky as the theoretical framework. Concept Maps can be useful in several different ways as are described in this paper. The use of concept maps in the TR-NYC approach are for the construction of the schemas of concepts that either students' or teacher-researchers are trying to grapple with, and this two-fold use of the concept map illustrates a part of the way the TR-NYC methodology works, from theory to practice and practice to theory, a bidirectional route.

1 Introduction

Understanding of concepts is a common theme of study across disciplines. In particular, the study of how concepts are understood is undertaken in mathematics education, mathematics, cognitive psychology, artificial intelligence, pattern recognition, and man-machine cognition. This article describes such a cross cutting approach that learns from each of the disciplines in the pursuit of increasing understanding of mathematics among students of remedial classes in the community colleges of the Bronx. The concept of fractions has unfortunately become a topic which students' young and not-so-young love to hate. A large-scale study (PROMYSE, 2006) consisting of 200,000 students in 60 districts across Ohio and Michigan reporting on the low passing rates of third through twelfth graders on a fractions test points out to some of the sources for the troublesome situation.

While mathematics in general and particular mathematics topics, including fractions are intriguing puzzles, their intrigue might remain inaccessible to students given their prior experiences in mathematics. Moreover, this "deficit" is detrimental as the PROMYSE study continues to draw our attention to: "*They are not learning enough to prepare them for the world they will face. They are not getting a chance to do all that they are capable of. In important ways, they are not making the grade even while they make their grades.* " (PROMYSE, 2006).

The role of the instruction in remedial mathematics courses at community colleges in this situation is to help students to re-construct, or in many cases, to construct the new schema of fractions. Generally, the schema is the network of the relationships between different components of the relevant concept, in this case, of a fraction. Once students are aware of the connections and of the procedures which participate in making these connections their knowledge can acquire a robust character, they "see" the fraction in its many manifestations.

In the NSF-ROLE #0126141 study (2002-2006), Introducing Indivisibles into Calculus Instruction, concept maps were envisioned as assessment tools, and this was based on the use of concept maps in science education as a research tool in the original work by (Novak and Gowin, 1984), although they were also seen before as chapter guides to some mathematical monographs written in the Bourbaki spirit. They are intended "as a graphical representation of the psychological structure of knowledge within the subject producing the map" (Novak and Gowin, 1984).

Concept maps discussed in this article represent a ready-made schema of general arithmetical relationships (Fig. 1), of Place Value (Fig. 3) or of the decimal fraction (Fig. 4). Our instructional task is to help students to understand these relationships and to make them their own, "to own them", to interiorize them. The question is how to do it?

In terms of the theory of instruction of Bruner, what one needs here is the outline of "how what one wishes to teach can be best learned, with improving rather than with describing learning." The theory of instruction needs to specify the ways in which a body of knowledge should be structured so that it can be most readily grasped by the learner, and it should specify the most effective sequence in which to present the material to be learned. We employ here Vygotsky's concept of the Zone of Proximal Development (ZPD) defined as the conceptual distance between what the student can understand by herself alone and that what she can understand with the help of the mentor, or peer discussions. The boundaries of student ZPD's are outlined by the spontaneous concepts of the learner, and the scientific concepts established by the profession, and the role of instruction is to create the conditions when the two can integrate with each other. The role of the instructor here is to choose the problems and questions, which, by a proper cognitive challenge help students to grasp needed relationships.

Hence, our attention has been focused on utilizing the power of concept maps, as ways of assisting learners in the construction of the schema of concepts from the bidirectional route:

- From spontaneous to scientific
- From scientific to spontaneous

Further, our use of concept maps is also to alleviate the difficult gap between procedural and conceptual understanding, which must be developed hand-in-hand.

In this article, the use of concept maps is demonstrated in the following ways:

1. as a means by which to provide students a snapshot or big picture,
2. as a way for teacher-researchers to design the problems in the instructional sequence (The structure of the concept map representation of the schema outlines the pedagogical design which will be taken in the course.)
3. as an environment within which analysis of word meaning can begin and progress toward a shared understanding (Bruner, 1990)

2 Facilitating Schema Development

In each case the process of meaning-making is facilitated by the use of the concept maps as pictorial scaffolding. Organization of mathematical knowledge which for students in college-level basic mathematics courses has been difficult is enhanced via the pictorial scaffolding of the concept maps. The concept map of the course in Figure 1 below serves as the organizing element for students throughout the semester. The concept map in Figure 2 serves a different purpose. Its aim is to aid the teacher-researcher in the design of the instructional sequence. Figure 3 has the aim of clarifying and reinforcing the process of working with numbers that involve decimal points.

The best way to discuss the schema development here is in the context of the procedural/conceptual divide known in mathematics education. There is a break between learning the procedure and understanding the concept this procedure relates to. Vygotsky's (1987) theoretical perspective offers clear solutions. All learners, especially students in remedial classes possess intuitive or "spontaneous" concept knowledge, which contains elements of different arithmetical procedures. On the other hand, the teaching environment has as its focus, the creation in the mind of the student, an understanding of the mathematical or "scientific" concepts. These latter concepts are those used by the profession as the working knowledge. Vygotsky's theoretical viewpoint as used in the TR-NYC methodology of Teaching-Research projects in the Bronx colleges takes on the investigation of creating the needed scaffolding from the diagnosed spontaneous knowledge to the required scientific via the Discovery approach (Czarnocha, Prabhu 2007a). Hence the possibility of integrating the spontaneous procedures with the mathematical, scientific concepts they are related to.

A Discovery-based development of instruction that progresses from spontaneous to scientific with clearly articulated conventions or embedded concepts used as basis by the mathematical community allows students to construct the desired concepts with validation at each stage of the mathematical truth of the concept in question, while one that misses the important middle step might lead to deep misconceptions that are difficult to clarify and which leave scars that have multiple repercussions. An example of such a mathematical concept is that of irrational numbers. An important hallmark of the irrational numbers was proposed by (Dedekind, 1901), viz., the Dedekind cuts. A Dedekind cut is

> *"If all points of the straight line fall into two classes such that every point of the first class lies to the left of every point of the second class, then there exists one and only one point which produces this division of all points into two classes, this severing of the straight line into two portions."*

The concept is difficult, often considered "higher mathematics" and as such out of scope of most elementary books of mathematics. However, many of those same expositions assume Dedekind's construction and a corollary, which while not explicitly stated is also not stated to be an assumption. The corollary states that there is a one-to-one correspondence between points on the geometric line and the set of real numbers. This assumption is difficult for students to comprehend. Hence, there appears in the exposition a jump or gap in the reasoning process. A logical step is missed and when this is not clarified, it creates in the mind of the learner a difficulty in navigating towards the conclusions that follow. Student schema of rational numbers contains then an unnecessary break in its structure. However, in the TR-NYC approach, when the appropriate connections, are explicitly articulated, students difficulty with the navigation of the concepts and understanding of the numbers of the number line significantly improve (Prabhu, Czarnocha, 2007).

Such an approach, integrating theory and practice on a continuous on-going basis requires that the failures/inadequacies in the constructions we had imagined would occur but did not, are open to our scrutiny and this scrutiny is quick, since the classroom of students needing attention, shortage of time, etc., continues without respite. So, if the needed help to students in their learning has to be provided then it is imperative that the teacher-researcher knows what to do, and knows what to do in short amounts of time. It is this blending of theoretical and practical perspectives that concept maps are very useful. The concept map provides a mediating tool with the help of which a student can be helped to develop the full concept she is studying. The analysis of meaning of words/concepts and of their connections to each other helps to create the model of the “scientific concept” in the student’s mind. Creation of such mental models is particularly suggested by ZPD framework of the Vygotskian theory; it forms the upper level of student ZPD’s, which, with the help of careful analysis of word meaning, and Bruner’s theory of instruction, explains all the mathematical conventions present in the map, to anchor that “scientific concept” in students’ minds.

Such a model is important in the context of students in community colleges of the Bronx because they are missing many components of the mathematical knowledge and linguistic knowledge and that absence makes it especially difficult to traverse the full developmental path by themselves.

3 Instruction

In the middle of the semester, close to the early part, a very bright student with a learning disability, questioned on seeing the concept map shown in Figure 1 below. “This is our syllabus”? After a few moments, in a very disbelieving voice, he asked again, “Are you serious, this IS our syllabus?” When enquired why he would have this question, he replied he had never imagined a syllabus as a picture. Cliches about pictures are well-known, however, the concept map serves an important role for students. It brings bring back the belief that Mathematics is not overwhelming, that it is not “a bunch of stuff to be memorized”, but rather that Mathematics is about big ideas deeply rooted in us which have stood the test of time, and are being scrutinized. Students and teacher-researchers together take up the concept map syllabus as a tool by which they

- navigate their teaching throughout the semester, altering the concept map as needed should such a need arise
- test their developing understanding
- communicate with each other in meaningful ways across a non-intimidating medium where words are few and sense can be made easier and quicker of what the other is trying to say.

The concept map in Figure 1, which appears on the first page of the Instructional Sequence, Story of Number (Prabhu, Czarnocha, 2007), serves the purpose of providing a snapshot of the schema of the concepts in question in the course. Knowing that a fraction is a number is an important new realization for some community college students. Understanding a fraction as division, as part of a whole, as decimal and percent asks a lot of the student, and such a map can help in grasping that concept through its many representations. The concept map is repeatedly visited throughout the semester. Thus, students used to seeing no connection between fraction, percent and decimal now have the capability of creating and seeing the created connections through classroom instruction. The underlying proportional reasoning inherent in the interrelated concepts takes on new meaning as a mechanism or tool whose use can be extended to situations where students would have had difficulty knowing how to begin the process of attempting a solution.

The concepts maps in Figures 2, 3, and 4, are utilized explicitly as schema building tools, directed primarily to students as conceptual scaffolding, and are examples for the development of students' schema of thinking. According to Skemp (1987), schematic learning provides a triple advantage over rote memorizing in that we are:

1. learning efficiently what we are currently engaged in
2. preparing a mental tool for applying the same approach to future learning tasks in the same field
3. when subsequently using this tool, we are consolidating the earlier content of the schema.

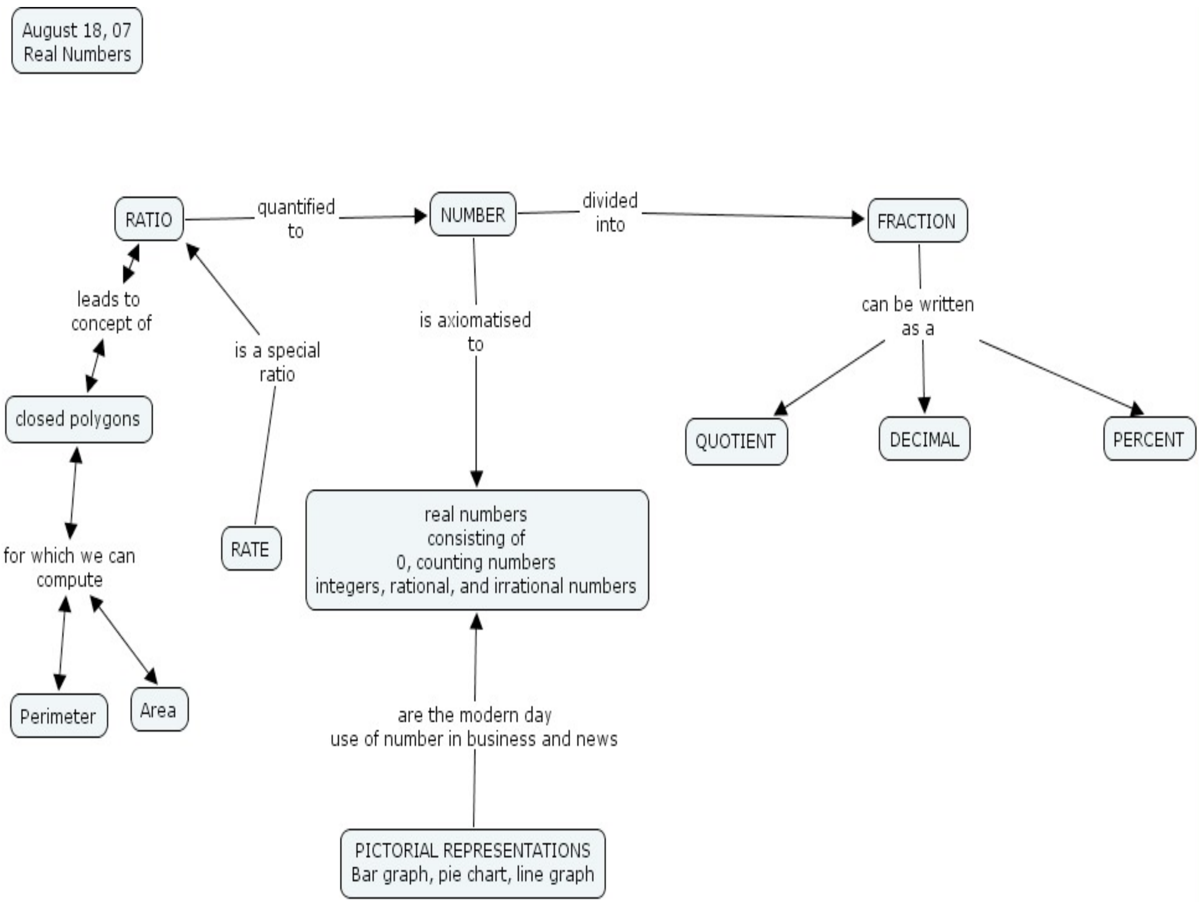

Figure 1. Concept Map that provides students a snap-shot of the course

The pictorial nature of the concept map as illustrated below assist in accomplishing all three points. Furthermore, the importance of learning based on the schema is stated by Skemp (1987) as follows:

A schema more than a concept, greatly reduces cognitive strain. Moreover in most mathematical schemas, all the contributory ideas are of very general application in mathematics. Time spent in acquiring them is not only of psychological value (meaning that present and future learning is easier and more lasting), but of mathematical value as well.

The concept maps below are thus to be viewed as the seeds of schema formation.

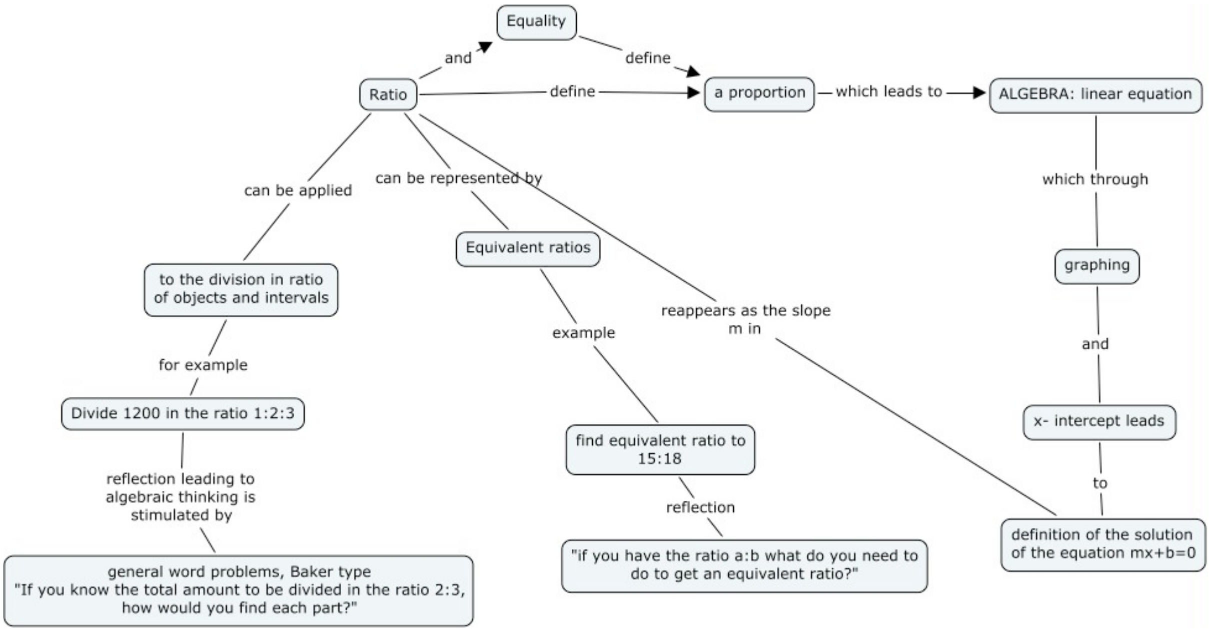

Figure 2. Methods of algebraization of the concept of a Ratio.

In the concept map above, the concept of the ratio, changes its role from a basic concept in the schema of a number (Fig.1) to its own subschema in (Fig.2), which indicates steps of fine level approaches to the algebraization of an arithmetic problem – one of the central themes of the arithmetic/algebra divide, which cause many problems both in schools and community colleges. Also a guide for the development of the instructional sequence, here it indicates the need for the separate concept of equality to be introduced, while at the same it is re-asserting its central position by reappearing at the end of one of the branches of algebraization process as the slope "m" of the line inherent in the equation. Its inner two branches provide a good scaffolding support for students developing their algebraic base of thinking. Can be used both as the scientific concept, that is its structure given to be explained by students or as the facilitator of relevant spontaneous concepts, which need to be developed through problem solving before they can be smoothly integrated with the scientific ones. In either case, conclusions can be drawn from students' responses about the process of schema development in an individual student's mind. Understanding of the details of that development can be helpful in refining the instructional approach for the mathematics classroom.

The Place Value concept map shows the connections between three different aspects of the Decimal System Notation: cycles of units, tens, hundreds, the powers of ten and place value. Color differentiation between the three concepts enhances the concepts differentiation while focusing attention on the connections between them wherever the color is changing. Designed especially for student use as the instructional tool around which to build the system and lead a class; it is very concrete and explicit in its content.

A third approach of the use of the concept map is when we wish to focus on the process of working with decimal points, another dreaded and misconception-laden topic. The following concept map in Fig. 3 below was found useful by students in two consecutive semesters.

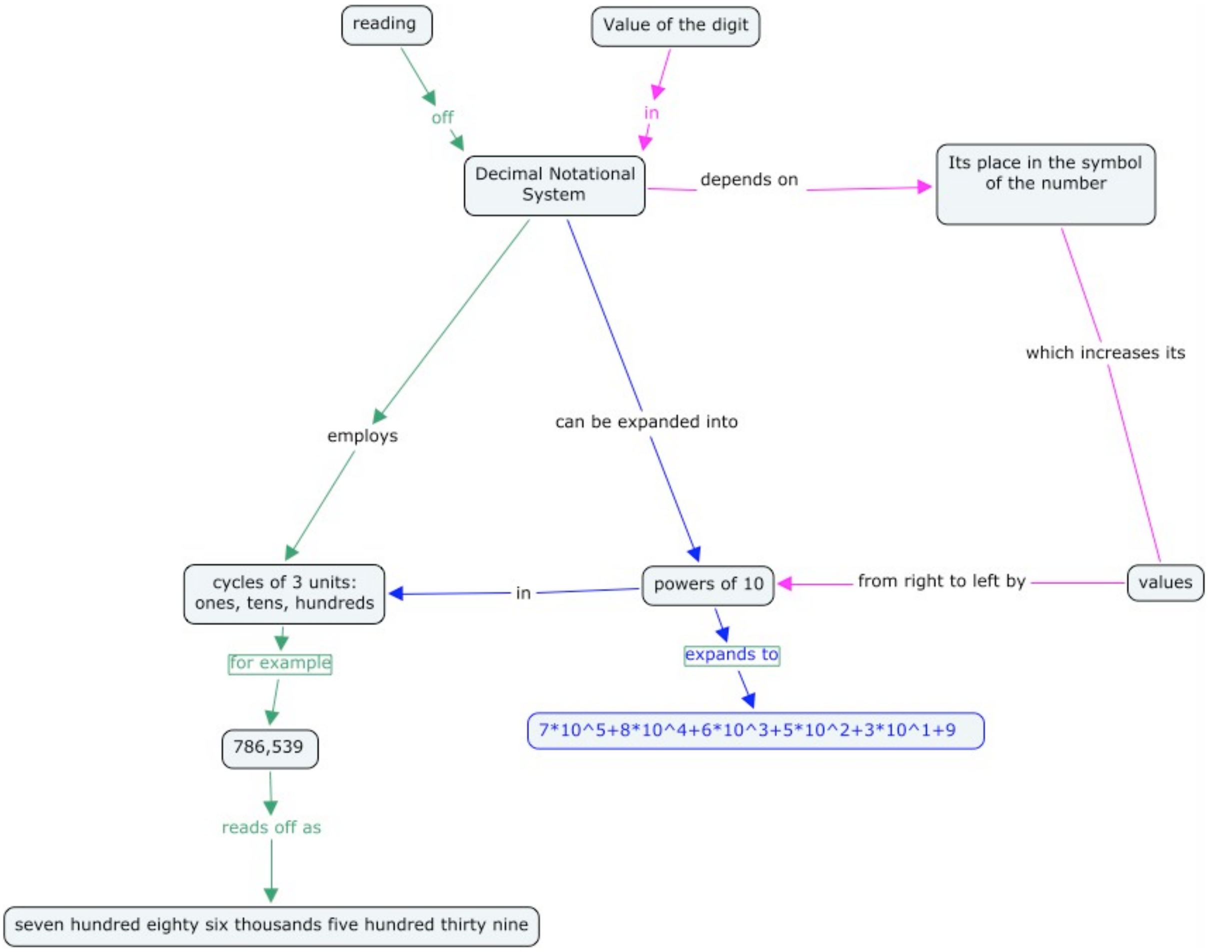

Figure 3. Place Value Concept Map

The Decimal Fraction concept map is designed in the similar style showing three different aspects of the decimal fraction: the decimal point, decimal expansion and the decimal alignment in addition or subtraction processes. General multiplicative technique of positioning the decimal point is absent since the understanding of its meaning is on a higher level of abstraction than its motion due to multiplication of the decimal fraction by powers of 10.

A fourth completely different approach appears in the creation of instructional sequences by the teacher-researchers of the team. It is known that students in the transition course from Arithmetic to Algebra have a high rate of attrition, i.e., about 67% of the students in community colleges at the City University of New York, who are registered in the Arithmetic course will not pass the following Elementary Algebra course. The needed transition, which assumes that students themselves can see how the algebra evolves from the arithmetic cannot be made. Hence, once again a scaffolding is required in which this transition is made explicitly clear, its explicitness is a regular discussion theme between teacher-researchers paying careful attention via language and other means, viz., concept maps to the transition points between Arithmetic and Algebra. The Instructional Sequence, Story of Number in Abstract (Czarnocha, Prabhu, 2008), in its evolving stages has utilized the following concept map in Figure 5. In the design of the instructional sequence, the concept map serves the purpose of being the mathematicians' palette similar to the artist's palette. Further, given the cyclical teaching-research methodology (Czarnocha, 2001), the concept maps used from semester to semester or from one Instructional Sequence version to another, demonstrate in their evolution the researchers' refinements in understanding the source of students' difficulties and their means of resolution via the appropriate mathematical intervention.

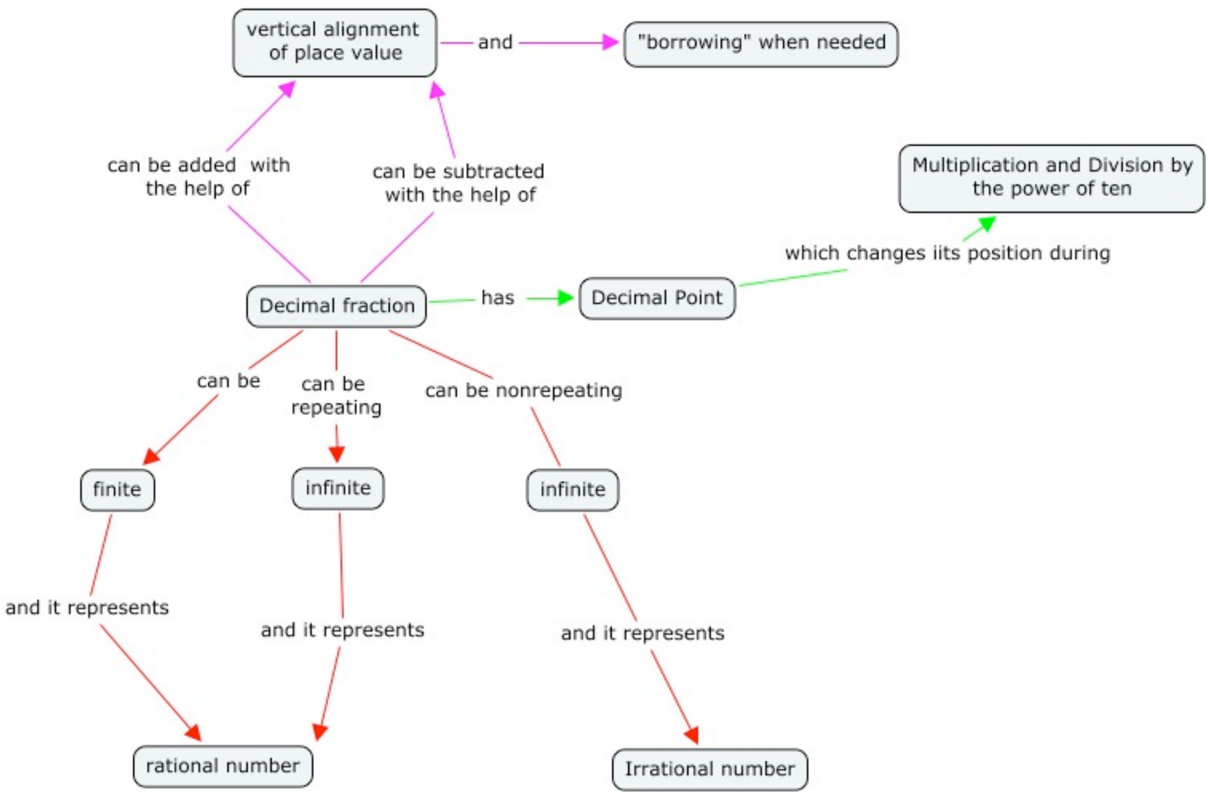

Figure 4. Decimal Fraction Concept Map

Given that (i) the concept map in Figure 5 below summarizes the entire course, (ii) arithmetic, the basis of algebra has been very troublesome to students in the past, (iii) English, the medium of instruction may not be the first language of many students, the concept map provides an opportunity to create meaning from analysis of a word or few words. Teacher-researcher and students can engage in a discussion of the few words within the concept map to establish understanding or to ferret out misunderstandings. Analysis of word meaning thus becomes an ongoing feature of the mathematics learning in question, and its starting point is the concept map.

4 Summary

Concept maps as used in our work accomplish the following

- facilitation of learning, via the integration of theory and teaching practice in the TR-NYC methodology of teaching-research;
- eliciting, capturing, archiving, and using "expert" knowledge, via the cyclical creation and refinement of instructional sequences for known to be difficult concepts;
- planning instruction via the use of the instructional sequence in actual classroom teaching;
- assessment of "deep" understandings via the periodic standard assessment instruments such as quizzes and tests and also the regular questioning in the classroom discourse;
- research planning via the cycles of teaching-research extending over several semesters and colleges.

Acknowledgements

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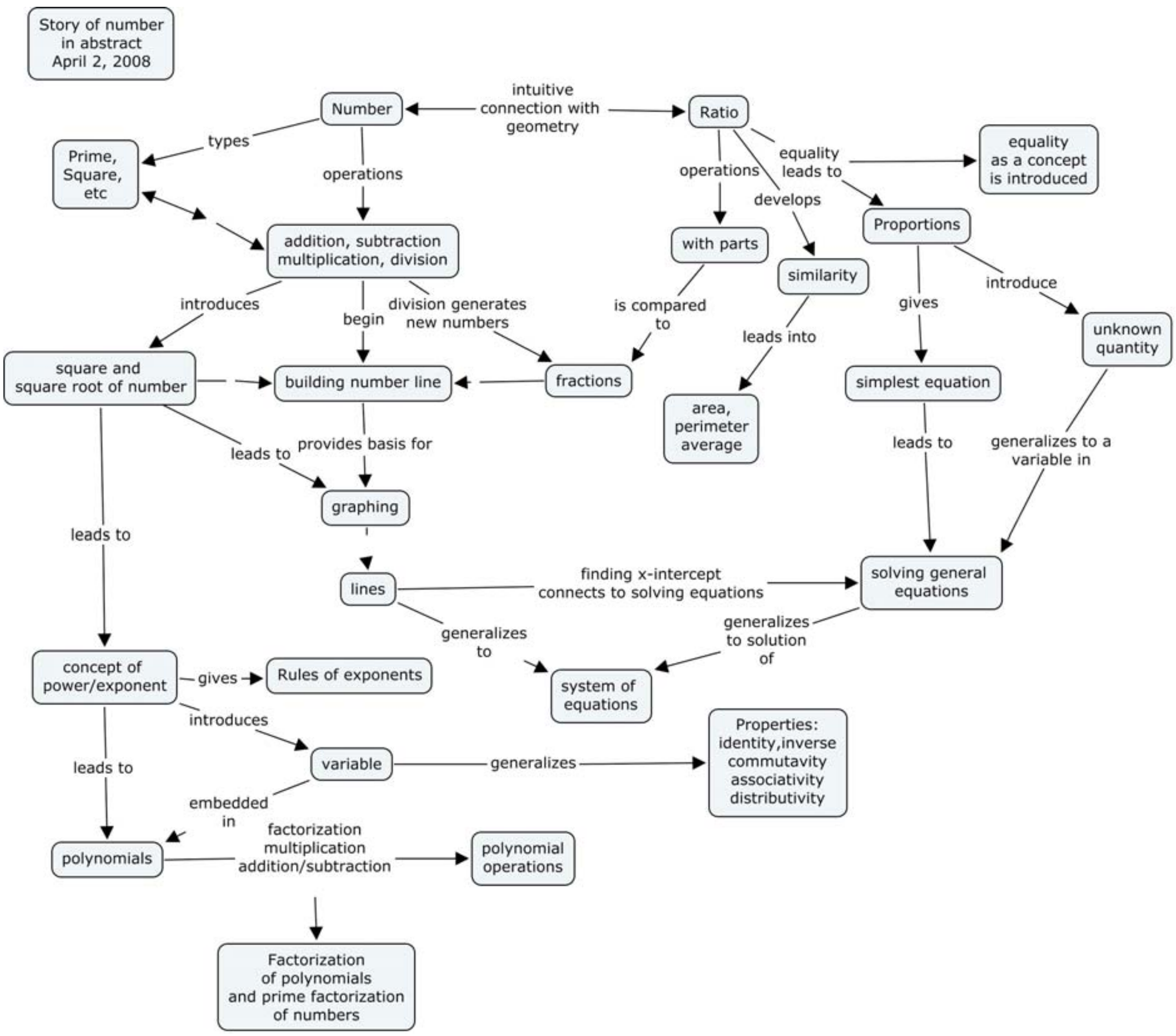

Figure 5. Preliminary Concept map to envision the design of the instructional sequence integrating arithmetic and elementary algebra.

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