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MAPPING CONCEPTS IN MATHEMATICS USING NETWORKS: THERE IS MORE INFORMATION IN MULTIPLE CHOICE ITEMS THAN YOU MIGHT THINK!

Geoff Woolcott, Southern cross University, Australia
Rassoul Sadeghi, University of New South Wales, Australia,
Daniel Chamberlain, Griffith University, Australia
Email: Geoff.Woolcott@scu.edu.au

Abstract. This paper investigates the advantages that may be obtained by mapping networks of connectivity between functional concepts in multiple-choice assessments. Current methods of analysis, for example, methods that use the Rasch model, are generally confined to analysis of linear connections of item responses and their corresponding outcomes. This paper examines multiple choice mathematics assessments using network analysis methods which examine interlinked connections between a broad assemblage of concepts, including outcomes, or learned concepts, and inherent item concepts derived from the assessment items. This paper outlines how network analysis, or other analysis that examines non-linear connectivity, may be useful in providing additional and useful information about multiple choice test items that complements the analysis provided using linear models.
Keywords: mathematics concepts, networks, connectivity, complex, mapping concepts.

1 Introduction

Multiple choice items are used widely for assessing mathematics learning and are a common test instrument in large-scale assessment programs. Analysis of the results of such assessment programs is generally used to compare student responses to items based on linear development of curriculum outcomes. Item response theory (IRT) underlies many methods of analysis utilised in examining multiple choice items, including methods based on the commonly used Rasch model (Rasch, 1960). Such methods allow calculation of a relationship between totals of item responses and ability estimates based on cohort performance (e.g., see Bond & Fox, 2007).

Feedback about mathematics learning from such multiple-choice testing is limited, however, in that it is generally based on comparing student responses to items based on curriculum outcomes. In Australia, for example, there is a strong focus on benchmarking and comparisons across and within cohorts (Lowrie & Diezmann, 2005). Multiple choice testing, however, may be able to provide more information for teachers than is available from Rasch analysis or that based on other IRT models, since such information may be obtained from items and items responses that describe the relationship between the concepts delineated by curriculum outcomes and other concepts that a student may have learned. Network analysis, a modern development linked to graph theory (e.g., Newman, Barabási, & Watts, 2006), enables items and responses to be described and treated in terms of interconnected links between learned concepts, including curriculum outcomes, and inherent item concepts (Woolcott, Chamberlain, Scott & Sadeghi, 2014). In order to illustrate how network analysis can be utilised to map such concept connectivity and provide additional information, this paper presents simple network representations and associated metrics related to multiple choice items in mathematics assessments for primary school Years 3 to 6 in NSW, Australia.

2 Background

2.1 Concept connectivity

Student performance in mathematics, as determined from formal assessments, may be associated with issues of connectivity related to interlinked concepts within mathematics (Mowat & Davis, 2010), as well as to linkages between concepts in mathematics and other subjects (e.g., Lakoff & Núñez, 2000). Connectivity of mathematics concepts may underpin the development of mathematics expertise, with student failures related to inadequate development of the interlinking of these concepts, for example in secondary algebra (Khattar, 2010). While issues of concept connectivity may act in a broad and complex system together with other issues, such as teacher quality and socioeconomic influences, (Morrison, 2003), examining some such connections in discrete entities, such as multiple choice test items, may be useful in informing educators working within the subject of mathematics (Woolcott et al., 2014).

Such connectivity illustrates a significant issue in learning within classrooms, as well as in designing successful mathematics interventions or revision, that relates to educators needing to understand how functional conceptual relationships are being made by students when they are learning mathematics. Specifically, educators need to know whether and how concepts are connected in such conceptual relationships. For example, current methods based in IRT, such as Rasch analysis, provide some of this information and can present results from sequences of tests over time, but such analysis is usually determined by assumptions of a simple connection between an item and a particular curriculum outcome. This paper, therefore, focuses on such outcomes, referred to here as learned concepts (see Devlin, 2007), but infers also connections between such outcomes and concepts inherent in the multiple choice items (Woolcott, 2013; Woolcott et al., 2014). These inferred connections may provide a rich addition to the IRT-based analysis, particularly since network analysis can be applied to individuals or to class cohorts, either in single years or across year groups.

2.2 Connectivity and networks

While there has been some acknowledgement that mathematics learning may be associated with concept connectivity, there have been few successful attempts to examine such connectivity using assessment data. There have been recent studies, however, that have linked student learning and performance in mathematics to the complex and non-linear connectivity of concepts that can be examined using modern developments in network theory (Newman et al., 2006). Mowat and Davis (2010), for example, working from within the assumptions of embodiment theory, have developed a theoretical approach to complex learning in mathematics using network theoretical approaches. Khattar (2010) has offered support to this approach, suggesting that mathematics knowledge may develop as complex networks and, if so, learning success may be dependent on the development of hubs in networks that support such non-linear conceptual development, and on the development of weak links that circumvent hub failures. The exploration of networks, and the connectivity of nodes within them, using empirical data, has developed rapidly in recent years, but although networks analysis has been applied widely in a number of differing disciplines, there has been little such application in studies of mathematics education (e.g., see discussion in Kop & Hill, 2008). Network analysis offers significant potential, however, largely because the rules governing the relationships within such networks remain independent of the nature of the subjects being linked (Newman et al., 2006). Woolcott et al. (2014) have applied this type of analysis to examine multiple choice mathematics items and responses both within and across year levels, and this set of analyses is being further explored and illustrated in this paper.

3 Methodology

Multiple choice items are used widely across the industrialized world as part of educational testing. In Australia, multiple choice items are utilised in the government administered Australian National Assessment Program - Literacy and Numeracy (NAPLAN) (ACARA, 2012) administered in Years 3, 5, 7 and 9, as well as in privately administered programs, such as the International Competitions and Assessments for Schools program, which includes the Australasian Schools Mathematics Assessment (ASMA) (EAA, 2012), administered from Year 3 in primary school through to the final years of senior high school.

As part of a broader study, *MathsLinks: Spatiotemporal Links in Mathematics Learning in Classroom and Online Environments*, multiple choice test items and student responses were examined for a sample of 249 NSW students who had completed a Year 3, 4, 5 and/or 6 mathematics student assessment in ASMA for the years 2009, 2010, 2011 and 2012. Items and item responses were compared across each year level and, since 32 of the students had completed all four of these tests in successive years (accounting for 128 of the above sample tests results), comparison was made also of longitudinal connectivity of items and item responses. Network analysis was used to examine the connections between outcomes and inherent item concepts in the multiple choice assessment items and responses. This paper presents, as an examplar, an analysis of only those items classified as Measurement according to the NSW K-6 Syllabus (BOS, 2012).

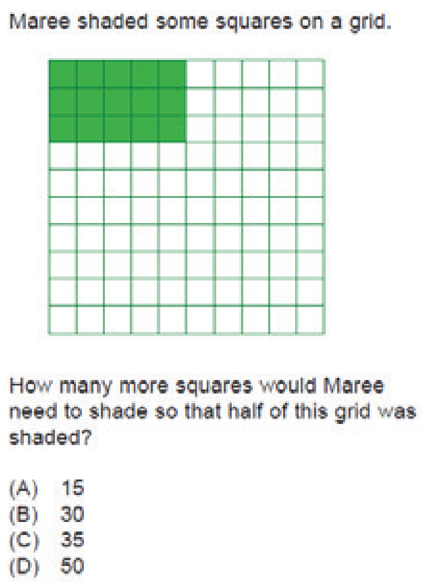
3.1 Network analysis

Network analysis, in particular place-based network analysis, allows qualitative mapping of the relationships between different nodes, here represented by outcomes, or learned concepts, and concepts inherent in items or, in the longitudinal analysis, by items. (The grouping of outcomes and inherent item concepts will be referred to collectively in this paper as ‘outcomes/concepts’). This style of analysis has been widely applied in different disciplines over the last two decades (Newman et al., 2006) and is applied in a novel way here in an educational context. In this study, coded data, generated by a concept survey, was subjected to a process of network analysis in order to produce graph representations and associated network metrics. An illustrative sample of network maps, published in more detail elsewhere, is used as a visual overview or snapshot of the mapped structure of the interconnectivity of the concepts and items under examination. This paper, however, focuses on the associated network metrics, which provide an extra dimension to interpretation of such network maps. Methodology for network representation is simplified here and follows Woolcott et al. (2014), but methodology related to network metrics for these measurement data is presented in more detail.

3.2 Concept survey and matrix coding

A matrix of coded data, generated from a concept survey of all measurement items, was analysed and maps generated using NetDraw (Borgatti, 2002). Each of the items was assigned one or more of the appropriate outcomes from the NSW K-6 Syllabus (BOS, 2012). Inherent item concepts, generated as 'access concepts' (Do I understand the question?) and 'answer concepts' (Can I now answer the question?), were developed by adapting Newman's Error Analysis (Newman 1977, 1983 in White, 2010). An example of concepts from this type of survey, derived from a published Year 5 ASMA practice question, is shown in Figure 1.

Measurement Outcomes (learned concepts)
MES1.2 Describes area using everyday language and compares areas using direct comparison
MS1.2 Estimates, measures, compares and records areas using informal units



Inherent item concepts

Access
Text and graphic
Recognise square
Recognise rectangle
Numeral represented as a symbol
Recognise objects with same area
Count objects
Recognise grid
Words shaded, some, squares, grid

Answer
Text and symbol
Interpret grid
Concept of fraction
Numeral represented as a word
Select informal unit to describe area
Estimate area using informal unit
Compare areas
Subtract using two numbers
Count squares
Words How many more, need to shade, half

Figure 1: Outcomes/concepts determined for a Year 5 multiple-choice measurement item. Practice item used with permission of EAA.

According to Newman's Error Analysis (NEA), there are five processes that a student has to negotiate in order to successfully answer the question posed by an assessment item; Reading (Decoding), Comprehension, Transformation, Process Skills and Encoding. Studies based on NEA indicate that a large majority of difficulties in answering mathematics questions may lie with the Comprehension and Transformation processes (Ellerton & Clarkson, 1996 in White, 2011). The access and answer concepts utilised here include a number of word and symbol representations inherent in items that enabled a focus on such processes. In many instances here, in fact, words are treated as concepts (e.g., see Radford, 2003), and the concept analysis uses overarching concepts that may allow interpretation of diagrams in multiple choice contexts (e.g., Lowrie, Diezman & Logan, 2012). A limitation in using NEA for multiple-choice items is that examination of student strategies, to do with Process Skills and Decoding, could not be used.

For each of the ASMA Years 3-6 measurement items, responses and survey results were coded as follows: correct items and associated outcomes/concepts as 1; and, incorrect items and associated outcomes/concepts as 0. In network maps constructed using the matrix, nodes are either concepts/outcomes or items. Table 1 shows a sampling of the coded matrix for a Year 6 student with Item 2 correct and Item 5 incorrect.

Table 1: Matrix coding for items and outcomes/concepts, for a Year 6 student with Item 2 correct and Item 5 incorrect (not all outcomes/concepts shown).

Outcomes/Concepts	Item 2	Item 2
MES1.5 (NSW K-6 outcome)	1	0
Recognise graph (access concept)	1	0
Read columns in graph (answer concept)	1	0
MS2.4 (NSW K-6 outcome)	0	0
Word: bought (access concept)	0	0
Adds mass in grams (answer concept)	0	0

3.3 Direct and inferred network connections

As well as direct relationships between concepts within single year groups, analysis utilised two types of inferred relationships: connections between all outcomes/concepts associated with an item; and, connections of all outcomes/concepts across two or more items where those items shared one or more outcomes/concepts. The network maps derived from these inferred relationships provide additional structural overviews of any concept connectivity. Two types of weightings have been calculated for these network connections: simple weighting based on total numbers of students with correct/incorrect item responses; and, weightings based on class averages of these item responses.

3.4 Network metrics

The following network metrics, *Centrality* and *Density*, indicated to be key metrics in studies of social networks (e.g., see Hanneman & Riddle, 2005), were applied to the undirected network data. *Centrality* was used to measure the degree to which network activity is centred on one or a few nodes or ‘actors’ (the network core), and provided insights into where influence may be concentrated, as well as blockages and patterns of flow across the network.

This study used three specific types of *Centrality*; *Connectivity* (*Degree)*, *Betweenness* and *Closeness*. *Connectivity (Average Path Distance)* is used to measure of the number of steps that it takes to navigate through each network. A specific connectivity measure, *Degree*, a count of the number of other nodes to which a given node was connected, was used here as a measure of strength of *Connectivity*. *Betweenness* is related to the number of connections between two nodes, but rather than measuring how close to the centre a node is, it measures how important the node is in traversing the network. *Closeness* is a measure of the distance of a node to all other nodes in the network calculated here by focusing on the most efficient (geodesic) distance from each node to all of the others in the network.

Density was used as a measure of cohesion, based on connectedness of dyads, or pairs of concepts, which are either connected or not connected. *Density* here refers to the number of connections compared to the total number of possible connections and this gives a measure, or index, of the connectedness of dyads within the population. This metric can be used to indicate clustering of concepts within the population, where the *Density* is higher than would be expected if the connections were random.

4 Results and discussion

4.1 Analysis based in IRT

Analysis based on IRT, such as Rasch analysis, can be used to provide an outline of student and class group achievement within a year level cohort, but can also be used to provide a view of student progress longitudinally across year levels. This widely used method of analysis can also give an overall picture of how a student is performing in terms of syllabus outcomes. Rasch analysis, however, can be used only to compare individual or class responses against the results obtained from a selected cohort who have done the same test at the same time under the same conditions. Longitudinally, such analysis can provide only a linear outline of progress in terms of outcomes, and indicate which outcomes that a student, in each year, may need to target in order to gain an improved performance in future mathematics assessments. The strength of this type of analysis is, in fact, in such an outline since this can suggest to a classroom teacher the items/outcomes that were difficult for individual students and for the class as a whole, compared within the class or compared with other cohorts.

4.2 Network analysis

4.2.1 Network representations within and across year levels

The Network Analysis utilised the concept survey to provide a representation of the connectivity between the outcomes (learned concepts) and concepts inherent in items (referred to here as access and answer concepts). This section shows mapped representations of connections for students in the same Year 5 school class as well as longitudinal connections for students in a Year 6 class who had completed all four assessments in the four successive years.

The network maps show both direct and inferred connectivity similar to that seen in Woolcott et al. (2014). In Figure 2, for example, direct connections (lines) between nodes represent items (squares) and their outcomes/concepts (circles) for incorrect items, are shown with connection weights calculated from totals of incorrect item responses across the Year 5 class. Figure 3 shows an inferred connectivity map for the same class, simplified by using only access concepts, based on those concepts that were shared across items. Inferred connectivity maps, such as sampled in Figure 3, may be used identify key outcomes/concepts that were, on average, incorrect/correct and that may/may not be a focus for teaching. This type of analysis may be used to represent connections between outcomes/concepts used correctly in one context and incorrectly in another, on average. The dotted square in Figure 3, for example, shows concepts shared across items that, although incorrect in one item, were correct in other items, in this case more than 50% of the time. As indicated in Woolcott et al. (2014), maps such as seen in Figures 2 and 3 indicate that particular measurement outcomes/concepts in this Year 5 assessment have not been understood by a number of students in this class, and may be a useful target for revision or intervention, even if only the centrally located outcomes/concepts are targeted. Based on this analysis, revision or intervention for either the entire class or for individuals could be designed around outcomes/concepts connected to incorrect item responses.

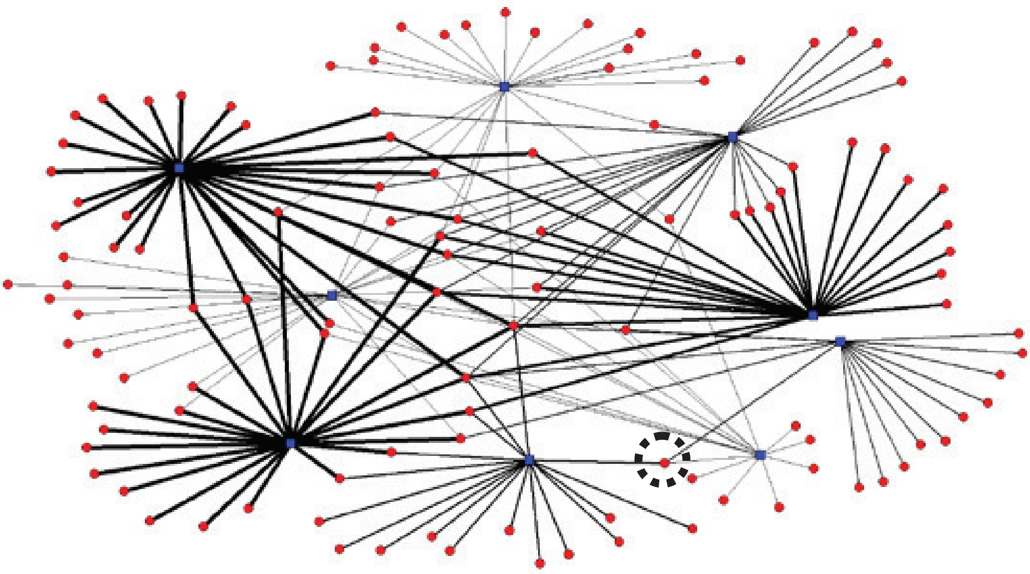


Figure 2: Direct connectivity map for Year 5 students with incorrect item responses - the heavier the line, the larger the number of incorrect responses. Items are indicated by filled squares and outcomes/concepts by filled circles. The dotted circle shows a shared concept.

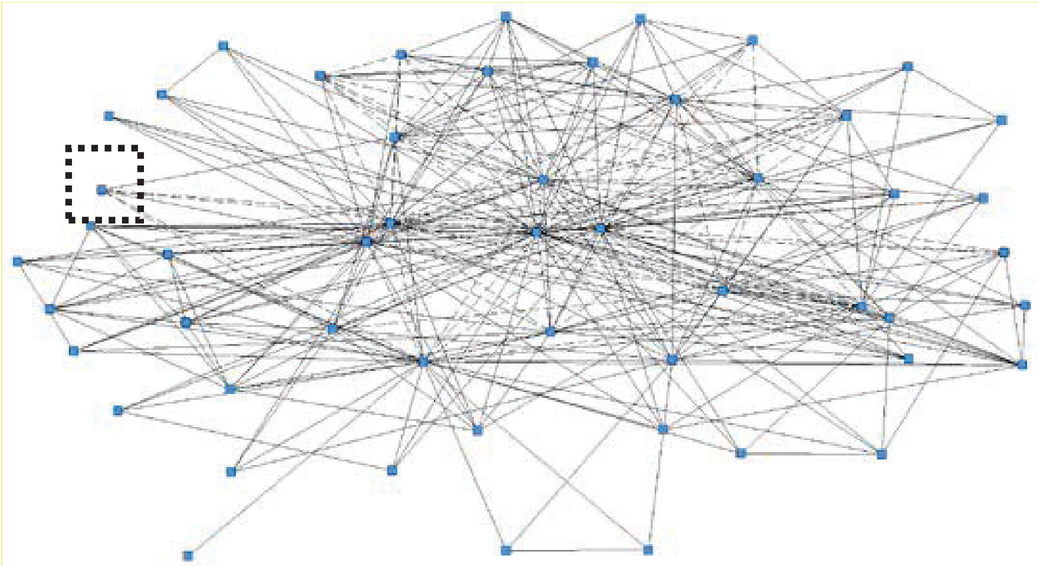


Figure 3: An inferred connectivity map, average weighted, for Year 5 students with incorrect item responses. Solid lines are based on incorrect to incorrect connections and dashed lines on incorrect to correct connections between inherent item concepts (with answer concepts not included for clarity). Concepts are indicated by filled circles. The dotted square (top left) shows a node that, on average, connects these concepts in incorrect and correct items.

An example of a representation of longitudinal connectivity is seen in Figure 4 based on inferred connections between the incorrect Item 20 in Year 6 and items in Years 3-5. The item connections were inferred from shared outcomes/concepts, effectively reversing the inference process utilised to construct Figure 3. For a focus on curriculum, this type of analysis can be adjusted to illustrate inferred connections between outcomes as well as between items, or connections between outcomes and inherent item concepts.

The effectiveness of the analysis used in Figure 4 lies in its illustration of the connection between Year 6 items and items in previous years, a concept map embracing past items, and indicating items and associated outcomes/concepts that were/were not learned successfully. Used in conjunction with the analysis represented in Figures 2 and 3, this type of analysis may be useful in designing revision or intervention that includes prior knowledge over time, as far back as Year 3 in this case. Two of the authors (Woolcott and Chamberlain) are using this longitudinal connectivity to trial an interactive online application which will link curriculum outcomes and inherent concepts to potential intervention strategies for both multiple-choice and other styles of assessment items, with the broader project aiming to test such strategies.

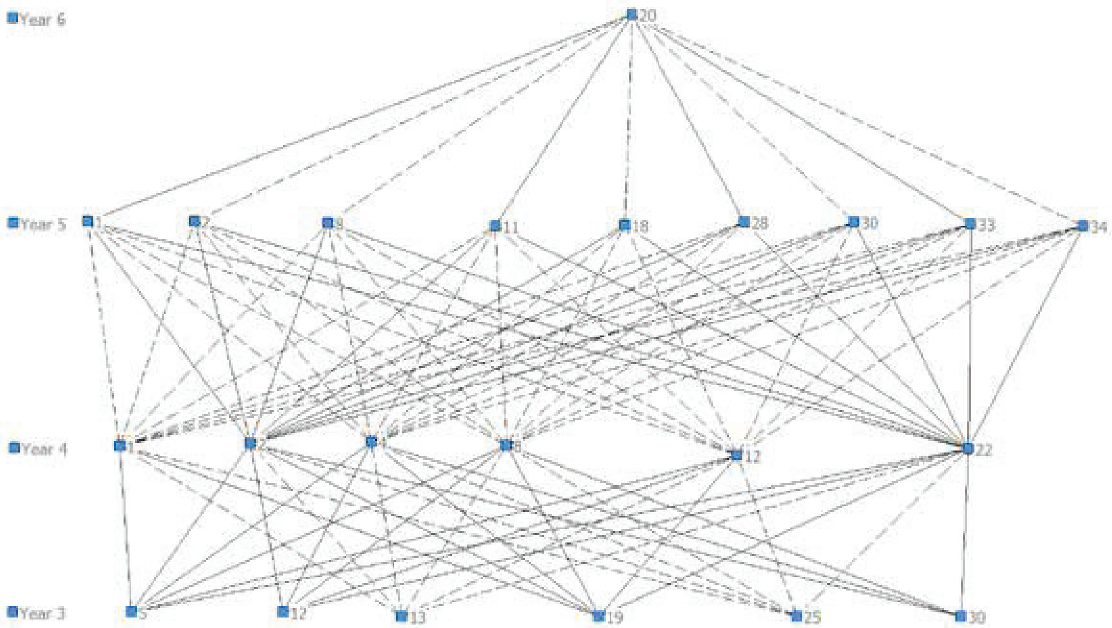


Figure 4: Inferred connectivity of the incorrect Year 6 Item 20 with items in Years 3-5. Solid lines are based on incorrect to incorrect connections and dashed lines on incorrect to correct items. Items are indicated by filled squares with an item number.

4.2.2 Network metrics

Table 2 shows examples of the network metrics applied to outcomes/concepts of the Year 5 measurement items. In the Year 5 data set, the mean for the *Degree*, the measure of *Connectivity* used here, of nodes in the network was 31.86 (all data here are rounded to two decimal places), with a standard deviation of 10.38. There were 17 outcomes/concepts that were more than two standard deviations from the mean, and this measure is used to indicate here that any node with a *Degree* greater than 34 may be significant. In theory, if a student is having difficulty with an outcome/concept, then the more significant the *Degree* of that outcome/concept the more likely it will have a negative impact on that student's understanding of the related outcomes/concepts. The higher the *Degree* that concept has, therefore, the greater potential there is for a negative effect.

Table 2: Examples of network metrics as applied to outcomes/concepts and their connections in Year 5 measurement items.

Node (outcome/concept)	**Concept type**	***Degree***	***Betweenness***	***Closeness***
Interpret grid	answer	17.59	0.31	54.82
Measure as word	answer	39.41	1.91	62.27
Numeral as number	access	85.67	18.64	87.46
one	access (word)	30.95	1.12	59.15
distance	access (word)	8.47	0	51.68

For this data set the mean *Betweenness*, which measures how important the node is in traversing the network, was 0.305, with a standard deviation of 1.51, indicating that any measure of *Betweenness* greater than 3.02 is significant. In this data set there were only 6 nodes judged on this basis as significant, and all of these were 3 or 4 standard deviations from the mean. A large number of significant *Betweenness* nodes would indicate that if a student is struggling making connections between outcomes/concepts, as indicated by such significance values, then designing revision or intervention that targets the concepts with the highest *Betweenness* values might be a priority. Remediation of these outcomes/concepts may assist a student in making connections both faster and more efficiently. The year 5 data set, however, is well connected and *Betweenness* is of limited use. This is not the case in other data sets, for example, the Year 6 data set (published elsewhere) has higher *Betweenness* values and many more nodes that are significant. For the Year 5 data set, the mean of the *Closeness*, which indicates how well connected the network is, was 52.04, with a standard deviation of only 4.32, indicating that a *Closeness* value of more than 61 was significant. There were only 13 nodes that judged on this basis to be significant and, therefore, well connected within the network.

The *Density* value calculated for this data set was 0.10 (308 connections out of 9812 possible, excluding the self loops), or just over 10% of all possible connections in the Year 5 measurement network, and this may be useful as a benchmark for students' incorrect answers and design of revision or intervention. A student, for example, whose item responses produced a network with a high *Density* value, relative to a benchmark, may be having significant issues across the board. If a student's network has a *Density* of 0.126, say, we can use this benchmark to say that their network has a relative *Density* of 0.845526, or alternatively that they are having troubles with 84.55% of the total concepts. This might be a useful way of prioritising the outcomes/concepts required for design of revision or intervention for particular students based on their functional understanding of what they have been taught.

It may be important to recognise nodes in the network that are more significant in one network metric than another. For example, the access concept 'measure as word' (Table 2) is highly connected with the rest of the network, as indicated by a significant *Degree* value, but it doesn't serve as a primary conduit for linking the network together since it has a non-significant *Betweenness* value. Some concepts in the Year 5 data set, however, are clearly significant in more than one network. The answer concept 'numeral as number' (Table 2), for example, is considered as signficiantly connected, and it appears to be, on *Betweenness* values, at the centre of connectivity pathways through the network.

5 Conclusion

This paper serves to illustrate the usefulness that network analysis may have in exploring concept mapping, in this case the spatiotemporal interconnectivity of mathematical concepts. The methodology used here draws on an extensive literature on networks that is only beginning to be applied in education (e.g., Kop & Hill, 2008; Mowat & Davis, 2010) as well as well-researched methods for examination of test items (e.g., White, 2010). There are, of course, limitations to using such methods in examining multiple choice items, not the least of which are the assumptions being made in nominating concepts as well as in inferring connections. The examples here, however, are part of an initial examination of whether network analysis is functional in the context of a school mathematics curriculum and the development of this project may offer resolution of such limitations. What this paper does show is that concept mapping can be explored using network analysis, and that such analysis may be useful in exploring conceptual development, not only in mathematics, but in any subject. The representation of longitudinal connectivity, in particular, gives a functional picture of conceptual development, in this case in mathematics, over time. This may have implications in education more broadly, particularly in relation to exploring how new knowledge, even when described in terms of outcomes, is based in prior knowledge as some cognitive psychologists suggest (e.g., see discussion in Sweller, van Merriënboer & Paas, 1998). Longitudinal representations of mapped concepts may allow a more extensive analysis of prior knowledge, however, than that undertaken in large-scale testing programs. The analysis here supports also broader analyses we are undertaking at differing conceptual levels, including analyses using embodied conceptualisations (Roth & Thom, 2009) and conceptualisations based on graphic elements in mathematics tasks (Lowrie et al., 2012) and pattern and structure (Mulligan, English & Mitchelmore, 2013).

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