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A PERFORMANCE SCORING METHOD BASED ON QUANTITATIVE COMPARISON OF CONCEPT MAPS BY A TEACHER AND STUDENTS

Makoto Takeya, Hitoshi Sasaki* , Keizo Nagaoka** and Nobuyoshi Yonezawa****
** Faculty of Engineering, Takushoku University*
815-1 Tatemachi, Hachioji, Tokyo 193-0985 JAPAN
{takeya, sasaki}@cs.takushoku-u.ac.jp
***School of Human Science, Waseda University*
2-579-15 Mikajima, Tokorozawa, Saitama 359-1192 Japan
****Faculty of Communication Engineering, Kogakuin University*
2665-1 Nakano, Hachioji, Tokyo 192-0015 Japan

Abstract. This paper discusses both a new measurement method and a performance scoring method in comparison with concept maps drawn by individual students with the concept map drawn by the teacher. However, it is very difficult to evaluate the degree of similarity between a pair of concept maps. Therefore, this paper presents a quantitative evaluation measure of similarity between a pair of maps by considering the difference of qualitative relations among these corresponding elements. This method has been effectively utilized to evaluate logical thinking ability.

1 Introduction

Lectures and texts are arranged in linear or sequential order. Each unit is presented in order. That is, they naturally move from one idea to the next, and so forth, without ever systematically detailing the structural relationships among these ideas. The teacher is concerned with assessing and promoting the acquisition of knowledge by individual students. Attention has recently focused on what has become known as 'structural knowledge' or knowledge of interrelationships among ideas in their knowledge domain. The need has been emphasized to establish the internal connectedness of ideas and concepts to be learned. It is difficult to evaluate these internal relationships among ideas by using traditional paper tests, because these tests mainly measure the understanding level of individual bits of knowledge obtained by individual students.

As one of the countermeasures related to the above problem, a graph-theoretical method of evaluating the knowledge is known as Shavelson’s graph-construction method. Shavelson and Stanton (1975) have reported the measurement of structural knowledge based on comparison between teacher and student's concept maps. The measurement provides a systematic method to help the teacher identify and communicate this structural information to students. An evaluation of differences between the student and teacher's structures provides insight into the student's organization of conceptual knowledge. It is very difficult to evaluate the degree of similarity between a pair of concept maps. Also, it is very difficult to measure a quantitative difference between a pair of maps.

Therefore, this paper presents both a new measure between a pair of maps and a performance scoring method based on concept maps drawn by individual students with the concept map drawn by the teacher.

2 A logical flow testing method by using the concept map

Next, let’s discuss a logical flow testing method by using the concept map to formative evaluation from different angles instead of the traditional formative test. The logical flow is a kind of a concept map, of which edges have single meaning of relationship, such as prerequisite relationship. The logical flow testing is testing in which individual students draw their concept map, called logical flow under restricted condition. A concept map can be represented by a digraph (directed graph) $G=(V,E)$, where V means a set of concept elements (vertices) and is composed of n elements (vertices), and E means a set of ordering relations (arrows) and is composed of m ordering relations (arrows). Here, the arrow $a \rightarrow b$ means ordering relation where element a is a necessary prerequisite to element b .

The concept map may be used as follows:

1) The teacher may draw the map $G=(V,E)$ according to the contents to be taught, and then teach the subject based on the map.
2) After lecturing on the subject, the teacher may give a logical flow test to draw each student’s concept map $G'=(V,E')$ with elements which are equal to those of the teacher’s concept map drawn in 1). Note that

arrow $\rightarrow$ has a unique meaning, *i.e.* one of prerequisite relation, cause and effect relation, influence relation, and so on. By this restriction, it is possible to measure qualitatively similarity degree between a pair of concept maps, in comparison with traditional Novak's maps (Novak & Gowin (1984)). The test can specify in advance that the concept map is composed of *n* concept elements (vertices), some elements included in *n* elements are initial concept(s) and a target concept and the meaning of ordering relation (arrow). Individual students have to draw their concept map, called their logical flows, arranging the rest concept elements and to drawing adequate ordering relations, *i.e.* arranging the rest vertices and to drawing adequate arrows.

3) The teacher can compare individual students' maps with the teacher's one, measure the structural concept levels of individual students, and evaluate their performance scores.

Note that individual vertices on each student's map are the same as ones on the teacher's map so that the teacher can evaluate how much individual interrelation of ideas that have been taught were established through the lectures. It follows that the discovery of their differences should be useful in conveying structural information to the student. A concept map drawn by the teacher serves as a teaching tool that allows the teacher to communicate interrelatedness of ideas in the knowledge domain to students. The concept map produced by the teacher presents students with a graphical synopsis of the structural relationships among ideas. Comparison of logical flow tests among their maps can supply the degree of the students' structural knowledge understanding that is missing in traditional tests. These discussions of concept maps allow students a review in conceptualizing their structural knowledge level. The students are able to consider their misconception of relationships among ideas. The teacher is also able to extract not only misunderstandings in structuring students' knowledge, but also deficiencies in communicating or teaching structural knowledge to students. In order to present our measurement of similarity between two cognitive maps, models of cognitive maps are introduced as shown in Figure 1 and Figure 2 (Takeya (1999)).

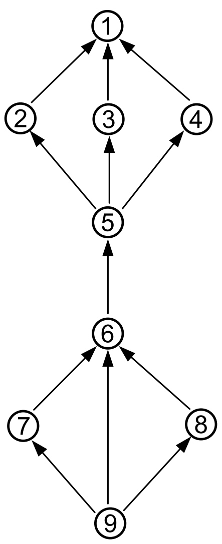

Figure 1 A model of the teacher's map

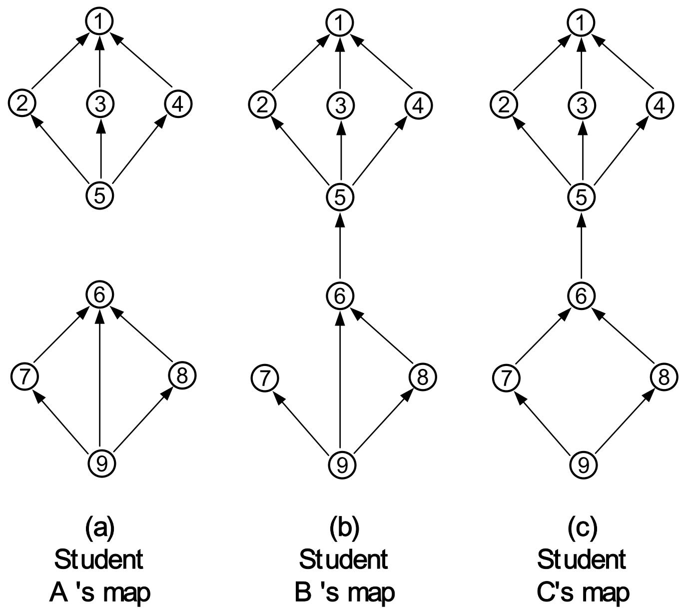

Figure 2 Models of three students' maps corresponding to the teacher's one in Fig.1

In **Figure 2**, pay attention to Three edges, (6, 5), (7, 6) and (9, 6), which all share the vertex 6. **Figure 2** shows three digraphs in which only one edge, (6, 5), (7, 6) or (9, 6), is lacking respectively when compared with the teacher's one shown in **Figure 1**. That is, concept maps of three students A, B and C respectively lack one relation, *i.e.* one edge as shown in **Figure 1**, compared with the teacher's one. From a view point of traditional scoring, it is considered that each student's performance score is equivalent to each other, because the numbers of lacking relations are equal to each other.

It is, however, intuitively thought that student A's map in **Figure 2 (a)** is most different from the teacher's one in **Figure 1**, because it is separated into two sub-digraphs. The elements from 6 to 9 can never reach the elements from 1 to 5. In **Figure 2 (b)**, there is no path from 7 to elements 1 - 5. On the other hand, one direct edge never exists only in **Figure 2 (c)**. Therefore, generally speaking, these maps are considered to be more similar to the teacher's one in the order of students C, B and A. Conversely speaking, edge (6, 5) makes the largest contribution to the teacher's map structure, and contrary to this edge (9, 6) makes the smallest contribution to the digraph. Next, this paper presents a new similarity measure between a pair of concept maps.

3 Similarity measure between a pair of concept maps

3.1 Importance degree of a relation

Here, let's define the ordered pair set $C(v_i, v_j; G)$, based on investigation of model comparisons among teacher's map and individual students' maps, shown in **Figure 1** and **Figure 2**. An ordered pair set $C(v_i, v_j; G)$ of the edge (v_i, v_j) is composed of different pairs of elements which are the starting and ending elements of paths through the edge (v_i, v_j). For example, in **Figure 1**,

C(6,5)={(6,5),(6,4),(6,3),(6,2),(6,1),(7,5),(7,4),(7,3),(7,2),(7,1),(8,5),(8,4),(8,3),(8,2),(8,1),(9,5),(9,4),(9,3),(9,2),(9,1)}.
C(7,6)={(7,6),(7,5),(7,4),(7,3),(7,2),(7,1),(9,6),(9,5),(9,4),(9,3), (9,2),(9,1)}.
C(9,6)={(9,6),(9,5),(9,4),(9,3),(9,2),(9,1)}.

The number of elements in the set relates to the contribution degree of the edge in digraph $G = (V, E)$. Next, define the degree of importance for a directed edge in digraph $G = (V, E)$. The importance index $I(v_i, v_j; G)$ of the edge (v_i, v_j) in $G = (V, E)$ is defined by the number of $C(v_i, v_j; G)$ normalized by the maximum number of $C(v_i, v_j; G)$. That is,

$$I(v_i, v_j; G) \equiv \frac{N[C(v_i, v_j; G)]}{Max.\{N[C(v_i, v_j; G]\}}$$

Here, $N[S]$ means the number of elements in set S .This definition can be formalized as follows:

$$I(v_i, v_j; G) \equiv \frac{N[A(v_i)] \times N[R(v_j)]}{[n/2] \times [(n+1)/2]}$$

Note that $A(v_i)$ and $R(v_j)$ means an ascendant set of v_i and a reachable set of v_j Here, the symbols of [ ] in the denominator are the Gauss marks. For any edge (v_i, v_j) in $G = (V, E)$,

$$0 \le I(v_i, v_j; G) \le 1$$

The values of $I(6,5; G)$, $I(7,6; G)$ and $I(9,6; G)$ in **Figure 1** are calculated as follows: $I(6,5; G) = 1.0$, $I(7,6; G) = 0.6$ and $I(9,6; G) = 0.3$,respectively. That is, $I(6,5; G)$ > $I(7,6; G) = 0.6$ > $I(9,6; G) = 0.3$. The edge (6, 5) is superior to the edge (7, 6), and the edge (7, 6) is superior to the edge (9, 6) in point of importance on $G = (V, E)$.

3.2 Comparison among concept maps

It is important to compare the concept map drawn by the teacher with those drawn by the students. However, the largest difficulty in comparison among these maps is to measure to what degree they are similar to each other from a quantitative standpoint. Considering the case where the teacher compares the teacher's map with some student's map as shown in **Figure 1**, the teacher must learn how to measure the similarities between them. Though a few measurement methods have been proposed to evaluate similarity among a set of maps, these methods involve a number of problems that must be solved. Shavelson (1972) has presented a quantitative method for map differences based on the total difference in number of input and output degrees among corresponding elements on two maps. One after the other, Shavelson and Stanton (1975) presented quantitative methods for the map difference according to the sum of distance margins of corresponding elements in two maps. The problem in these methods lies in the fact that no consideration was given for differences in qualitative relations among corresponding elements between the two maps. In order to solve the above problem, Takeya and Sasaki (1997) presented a quantitative evaluation measure of difference between a pair of maps by considering the difference of qualitative relations among these corresponding elements. Based on the distance similarity, a similarity $S(G, G')$ between two digraphs $G = (V, E)$ and $G' = (V, E')$ is defined as follows:

$$S(G,G') \equiv \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} N[C(v_i,v_j;G \cap C(v_i,v_j;G')]}{\sum_{i=1}^{n}\sum_{j=1} N[C(v_i,v_j;G \cup C(v_i,v_j;G')]}$$

It can be interpreted as the degree of common parts among the importance degrees between all pairs of edges on both $G=(V,E)$ and $G'=(V,E')$. On the digraphs $G=(V,E)$ and $G'=(V,E')$, the following statements are equivalent to each other. (1) $S(G,G')=1$, and (2) $G=G'$.

3.3 Performance scoring method

In general, let $R(G';G)$ denote the performance score for the student who drew the concept map G', and suppose the teacher drew the ideal G. Then, the performance score $R(G';G)$ for the student is defined by comparing the student map G' with the ideal map G drawn by the teacher.

$$R(G';G) \equiv 100\sqrt{S(G,G')}\ .$$

Let us suppose that map G_T was drawn by the teacher and the others G_A, G_B and G_C were drawn by students A, B and C in **Figure 1**. Then, their performance scores are calculated. These values can be tabulated in Table 1. As is evident from this table, $R(G_A;G_T) < R(G_B;G_T) < R(G_C;G_T)$. These results agree with the ones on degrees of importance calculated. The details of the utilization method and the results of our case study will be explained on the poster of the day.

4 Summary

This paper discussed a new quantitative measurement and formative evaluation methods based on comparison among teacher's concept map and individual students' maps. This method has been utilized for formative evaluation in Japan and Taiwan.

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